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Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Frederick Hurn Constable, M.A., Ph.D., D.Sc., St. John's College, Cambridge (proposed by H. Jeffreys) ;

Theodore Dunham, Jr., Ph.D., M.D., Mount Wilson Observatory, Pasedena, Cal., U.S.A. (proposed by W. S. Adams) ;

Alfred L. Loomis, Loomis Laboratory, Tuxedo Park, New York, U.S.A. (proposed by H. H. Turner) ; and

Andrew Cecil Telfer, M.A., Dinham Hall, Ludlow, Salop (proposed by M. Proctor).

Sixty-nine presents were announced as having been received since the last meeting, including, amongst others :—

Harold Jeffreys, The Earth, its Origin, History, and Physical Constitution (2nd edition).

Sir Joseph Larmor, Mathematical and Physical Papers (2 volumes).

Rev. Thomas Nicklin, Planetary Calculations for Dates in the First Century B.C. (MS.).

Specola Vaticana : Astrographic Catalogue, vol. x.

The Perturbations of Comet Pons-Coggia-Winnecke-Forbes from 1818 *to* 1928. By A. C. D. Crommelin.

The comet discovered by Mr. A. F. I. Forbes at Capetown on 1928 November 19* proved to be identical with comet 1818 I (Pons) and 1873 VII (Coggia-Winnecke). The identity of these two had already been suspected by F. Argelander (*A.N.*, **82**, 381), E. Weiss (*A.N.*, **87**, 122), and L. Schulhof (*B.A.*, **3**, 125, 173, 265), but it could not at that time be definitely established owing to the shortness of the arcs of observation, which were 4 days in 1818 and 5 days in 1873; also, assuming identity, it was quite uncertain how many revolutions the comet had made between the two apparitions. Fortunately it was observed for a month in 1928, which has enabled the period to be deduced as about 28 years, so that it has completed four revolutions since 1818. It is the only comet, except Halley's, outside the Jupiter and Saturn groups that has been observed at three apparitions, and it is of interest to examine whether planetary perturbations will explain the inequality in the intervals, which are 55 years 10 months and 54 years 11 months respectively. It was for this purpose that I undertook the following researches. I give for reference the approximate elements at the three apparitions, which, for convenience, I have reduced to the equinox of 1873. We shall see later that the observations of 1818 were too rough to enable good elements to be deduced from them; those given are by Schulhof. Schulhof gave a series of different elements for the 1873 apparition, but I only give the ones corresponding to the period 27·91 years. Those for 1928 III are by H. E. Wood.

Comet.	T.	ω.	Ω.	i.	log q.	Period.
		° ′	° ′	° ′		y
1818 I Pons	Feb. 4·66	195 18	250 29	25 39	9·87592	9·375
1873 VII Coggia / Winnecke	Dec. 1·93273	196 7·6	249 48·7	28 48·0	9·87330	27·910
1928 III Forbes	Nov. 4·493	195 53·9	249 22·5	28 54·8	9·87216	28·655

T is given in G.M.T. (Old Style).

The investigation of the perturbations was made on the lines of my paper in *Memoirs*, vol. lxiv, but as the eccentricity 0·91838 falls outside the limits given there, it was necessary to calculate the coefficients I to X.

The interval of M was taken as 9°, which corresponds to about 250 days in a period of 28 years; this is too large to justify the expectation of very high precision in the results, but it suffices to establish the fact that the perturbations both in the period and in the other elements are very close to those required for identity. The computation is simplified by the fact that there have been no very close approaches to planets during the whole period. The following are the minimum distances in

* It was also glimpsed by Mr. Yamasaki at Misusawa Observatory on 1928 October 26.

astronomical units, with the approximate dates. Jupiter, 5·1 in 1818, 4·8 in 1843, 4·5 in 1846, 2·6 in 1875, 4·9 in 1901, 4·9 in 1926, 4·4 in 1928. Saturn, 2·9 in 1842, 3·7 in 1870, 6·0 in 1898, 8·1 in 1927. Uranus, 10 in 1824, $9\frac{1}{2}$ in 1907. Neptune, $17\frac{1}{2}$ in 1830. It may be of use to other computers to tabulate the values of v, $\log \frac{r}{a}$, and the logarithms of the coefficients I to X, which include the logarithm of Jupiter's mass. The numerical values are the same for argument 360° − M, the signs being reversed in columns marked R.

M.	v. R.	$\log \frac{r}{a}$.	log I. R.	log II.	log III. R.	log IV.
°	° ′					
0	0 0	8·9117	..	0·8048	..	1·7915
9	130 34	9·5896	1·3634	1·2959*n*	1·3711	9·6754
18	144 50	9·7980	1·4516	1·6037*n*	1·2509	1·1060*n*
27	151 42	9·9128	1·4818	1·7507*n*	1·1663	1·3246*n*
36	156 8	9·9902	1·4904	1·8415*n*	1·0975	1·4399*n*
45	159 23	0·0473	1·4870	1·9118*n*	1·0370	1·5163*n*
54	161 58	0·0919	1·4757	1·9631*n*	0·9811	1·5723*n*
63	164 7	0·1277	1·4581	2·0039*n*	0·9278	1·6155*n*
72	165 58	0·1571	1·4351	2·0370*n*	0·8754	1·6502*n*
81	167 35	0·1816	1·4069	2·0644*n*	0·8227	1·6785*n*
90	169 4	0·2021	1·3735	2·0873*n*	0·7686	1·7019*n*
99	170 24	0·2195	1·3344	2·1065*n*	0·7122	1·7215*n*
108	171 40	0·2341	1·2887	2·1226*n*	0·6519	1·7378*n*
117	172 50	0·2464	1·2353	2·1361*n*	0·5863	1·7514*n*
126	173 57	0·2566	1·1722	2·1473*n*	0·5129	1·7626*n*
135	175 2	0·2649	1·0961	2·1564*n*	0·4284	1·7718*n*
144	176 4	0·2715	1·0016	2·1637*n*	0·3273	1·7791*n*
153	177 4	0·2766	0·8785	2·1692*n*	0·1992	1·7846*n*
162	178 3	0·2801	0·7037	2·1730*n*	0·0209	1·7884*n*
171	179 2	0·2822	0·4033	2·1753*n*	9·7184	1·7907*n*
180	180 0	0·2829	..	2·1761*n*	..	1·7915*n*

M.	log V.	log VI. R.	log VII.	log IX. R.	log X.
°					
0	1·1247	..	9·3506	..	1·8493*n*
9	0·9379*n*	1·5473	1·4390*n*	1·4101*n*	1·1714*n*
18	1·0372*n*	1·5852	1·6352*n*	1·2898*n*	0·9629*n*
27	1·0695*n*	1·5948	1·7424*n*	1·2052*n*	0·8482*n*
36	1·0859*n*	1·5918	1·8150*n*	1·1364*n*	0·7708*n*
45	1·0960*n*	1·5811	1·8688*n*	1·0760*n*	0·7137*n*

M.	log V.	log VI. R.	log VII.	log IX. R.	log X.
°					
54	1·1029*n*	1·5645	1·9109*n*	1·0201*n*	0·6691*n*
63	1·1078*n*	1·5430	1·9449*n*	0·9668*n*	0·6333*n*
72	1·1116*n*	1·5170	1·9729*n*	0·9143*n*	0·6039*n*
81	1·1145*n*	1·4865	1·9962*n*	0·8616*n*	0·5794*n*
90	1·1168*n*	1·4512	2·0159*n*	0·8076*n*	0·5589*n*
99	1·1186*n*	1·4105	2·0324*n*	0·7511*n*	0·5415*n*
108	1·1201*n*	1·3637	2·0465*n*	0·6909*n*	0·5269*n*
117	1·1213*n*	1·3092	2·0582*n*	0·6252*n*	0·5146*n*
126	1·1223*n*	1·2453	2·0680*n*	0·5518*n*	0·5044*n*
135	1·1231*n*	1·1685	2·0760*n*	0·4674*n*	0·4961*n*
144	1·1237*n*	1·0735	2·0823*n*	0·3663*n*	0·4895*n*
153	1·1242*n*	0·9500	2·0872*n*	0·2382*n*	0·4844*n*
162	1·1245*n*	0·7750	2·0906*n*	0·0598*n*	0·4809*n*
171	1·1247*n*	0·4744	2·0926*n*	9·7573*n*	0·4788*n*
180	1·1247*n*	..	2·0933*n*	..	0·4781*n*

log VIII = log VI + 9·5973*n*.

Since the value of n at any moment is unknown at the start, the perturbations in the first two revolutions were computed using the period 27·91 years, which is half the interval between the perihelia of 1818 and 1873; those in the other two revolutions with the period 27·46 years. The following tables give the perturbations thus found for the different elements.

	Perturbations of ω.				Perturbations of Ω.			
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.
Jupiter	− 264″·0	+247″·6	−553″·8	−191″·8	− 481″·0	+333″·3	− 900″·9	−627″·1
Saturn	+1013·8	+461·1	+120·8	+ 36·6	−1033·0	−370·3	− 153·9	−125·7
Uranus	+ 9·7	+ 2·5	+ 2·4	+ 15·0	− 4·6	− 1·2	+ 0·6	− 9·3
Neptune	+ 12·2	− 2·5	+ 1·3	+ 3·3	− 11·9	+ 2·1	− 0·4	− 2·9
Sum.	+ 771·7	+708·7	−429·3	−136·9	−1530·5	− 36·1	−1054·6	−765·0

	Perturbations of *i*.				Perturbations of φ.			
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.
Jupiter	− 322″·1	− 13″·5	+915″·6	−403″·5	+ 8″·3	+ 10″·2	−57″·8	+12″·6
Saturn	− 701·9	−242·6	− 91·5	− 43·2	+123·3	+ 91·5	+12·3	+ 6·4
Uranus	− 20·8	+ 0·6	− 4·3	− 28·9	− 13·1	+ 4·0	− 0·3	−17·1
Neptune	− 18·5	+ 3·2	+ 1·8	− 4·5	− 10·4	+ 2·3	+ 2·4	− 2·3
Sum.	−1063·3	−252·3	+821·6	−480·1	+108·1	+108·0	−43·4	− 0·4

	Perturbations of M_0.				Perturbations of n.			
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.
Jupiter	+2045″·1	+1697″·5	+2757″·5	+2042″·0	−1″·3958	+0″·3858	+2″·5286	−1″·6774
Saturn	+ 249·6	− 67·3	− 131·7	− 161·1	−0·1351	+0·0213	−0·0327	−0·0119
Uranus	+ 18·3	− 20·8	+ 21·5	+ 14·6	−0·0089	+0·0021	−0·0013	−0·0116
Neptune	− 8·5	+ 5·5	− 7·7	− 6·4	−0·0039	+0·0009	+0·0009	−0·0012
Sum.	+2304·5	+1614·9	+2639·6	+1889·1	−1·5437	+0·4100	+2·4955	−1·7021

	Perturbations of n (Second Summation).				Second Summation, combining two Revolutions.	
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Revs. 1 and 2.	Revs. 3 and 4.
Jupiter	+368″·1	−2016″·5	+27308″·5	−721″·7	−15863″·2	+51946″·8
Saturn	−835·1	− 252·8	+ 120·7	+526·6	− 2467·9	+ 320·0
Uranus	− 61·9	+ 1·7	− 7·2	− 82·9	− 150·9	− 103·5
Neptune	− 28·4	+ 8·0	+ 0·3	− 9·7	− 60·5	− 0·4
Sum.	−557·3	−2259·6	+27422·3	−287·7	−18542·5	+52162·9

Hence the total perturbation of M, which is found by adding the perturbation of M_0 to the result of the second summation of the perturbation of n, is $-14623''\cdot1$ for revolutions 1 and 2, and $+56691''\cdot6$ for revolutions 3 and 4.

It will save time to anticipate here the result of a re-examination of the observations of 1818, which indicates 1818 February 6·52 as a closer approximation to the date of perihelion than that given above (a still later value is February 6·30, but as it is uncertain by 0·25 it is unnecessary to change the figure).

The Julian days for the three perihelia are 2385107·52, 2405494·933, 2425555·493. From the first two, starting from perihelion in 1818, we have the equation

$$0 + n_{1818} \times 20387\cdot41 - 14623''\cdot1 = 2592000''.$$

The right-hand side is twice the number of seconds in the circumference. We thus obtain $n_{1818} = 127''\cdot8546$; period 1818 = $27^{y}\cdot7518$. Applying the perturbation in n, we find $n_{1873} = 126''\cdot7209$; period 1873 = $28^{y}\cdot0000$.

From the second two perihelia we obtain the equation

$$0 + n_{1873} \times 20060\cdot56 + 56691''\cdot6 = 2592000''.$$

We find $n_{1873} = 126''\cdot3827$; period 1873 = $28^{y}\cdot0749$; hence $n_{1928} = 127''\cdot1761$; period 1928 = $27^{y}\cdot8998$.

There is thus a discordance of $0^{y}\cdot0749$, or $27\frac{1}{3}$ days, between the two values found for the period in 1873. On examination I was led to ascribe this to the fact that the initial assumption of uniform period from 1818 to 1873, and from 1873 to 1928, gave erroneous times for the intermediate perihelion passages; hence the Jupiter and Saturn perturbations about these times need revision. I found that the 1846

perihelion was more affected in this way than the 1901 one. I provisionally adopted 28ʸ·05 as the period in 1873, and deduced the perihelion dates 1845 October 5 and 1901 April 26. These are respectively 90 and 23 days earlier than those used in the first calculation. (The revised calculation gives the dates of perihelion 1845 October 18 and 1901 May 7.) The perturbations by Jupiter and Saturn have been recalculated from the aphelion of 1831 to that of 1859, and those by Jupiter from the aphelion of 1887 to that of 1915.

The following table gives the revised values: revision was considered unnecessary for Uranus and Neptune, or for Saturn in the second pair of revolutions.

	Perturbations of ω.				Perturbations of Ω.			
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.
Jupiter	−227″·2	+251″·0	−554″·7	−194″·2	− 625″·3	+336″·5	− 905″·6	−632″·5
Saturn	+978″·4	+458″·1			− 953″·7	−368″·8		
Sum. of 4 planets	+773″·1	+709″·1	−430″·2	−139″·3	−1595″·5	− 31″·4	−1059″·3	−770″·4

	Perturbations of i.				Perturbations of ϕ.			
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.
Jupiter	− 353″·9	− 1″·2	+913″·0	−408″·4	+ 50″·1	+ 24″·6	− 78″·4	+18″·6
Saturn	− 648″·7	−233″·8			+128″·7	+ 96″·3		
Sum. of 4 planets	−1041″·9	−231″·2	+819″·0	−485″·0	+155″·3	+127″·2	− 64″·0	+ 5″·6

	Perturbations of M_0.				Perturbations of n.			
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.
Jupiter	+2083″·6	+1763″·9	+2799″·9	+2040″·9	−1″·5655	+0″·3225	+2″·5331	−1″·6536
Saturn	+ 182″·3	− 61″·1			−0″·0933	−0″·0777		
Sum. of 4 planets	+2275″·7	+1687″·5	+2682″·0	+1888″·0	−1″·6716	+0″·2478	+2″·5000	−1″·6783

	Perturbations of n (Second Summation).				Second Summation, combining two Revolutions.	
	Rev. 1.	Rev. 2.	Rev. 3.	Rev. 4.	Revs. 1 and 2.	Revs. 3 and 4.
Jupiter	+220″·0	−2677″·6	+27276″·5	−471″·7	−18303″·3	+52212″·8
Saturn	−782″·0	− 340″·7			− 2075″·0	
Sum. of 4 planets	−652″·3	−3008″·6	+27390″·3	− 37″·7	−20589″·7	+52428″·9

The first two revolutions give us, in the same manner as before, $n_{1818} = 127''{\cdot}9525$, $n_{1873} = 126''{\cdot}5287$; period 1818 = 27ʸ·7305, period 1873 = 28ʸ·0426.

The last two revolutions give $n_{1873} = 126''{\cdot}3677$, $n_{1928} = 127''{\cdot}1894$; period 1873 = 28ʸ·0783, period 1928 = 27ʸ·8969.

The discordance between the two determinations of the period in 1873 has now been reduced to $0^{y}\cdot0357$, or 13 days. It is presumably due to the 250-day interval employed in the perturbations being too long. It would have been better to have used intervals of 9° of *eccentric anomaly*, instead of 9° of *mean anomaly*; but I did not notice this till it was too late to retrace my steps.

It is worth noting that the unperturbed discordance is $5\frac{1}{2}$ months *in the other direction*, so that the calculation has over-explained it in the proportion of 14 to 13.

I pass now to the discussion of Pons's observations in 1818, and a comparison between them and the positions computed from the following assumed orbit, which I deduced by applying the perturbations found above to L. Schulhof's elements for 1873.

$$\left.\begin{array}{rrr} \omega & 195^{\circ} & 43'\cdot8 \\ \Omega & 249 & 29\cdot1 \\ i & 29 & 9\cdot5 \end{array}\right\} 1818\cdot0.$$

$$\begin{array}{rrr} \phi & 66 & 43\cdot2 \end{array}$$

$$\log q \quad 9\cdot87265$$

$$\text{Period} \quad 27^{y}\cdot731$$

Gaussian equations:

$$\begin{aligned} x &= [9\cdot94931]\, r \sin (v + 172^{\circ}\, 32'\cdot2) \\ y &= [9\cdot99192]\, r \sin (v + 76\ 48\cdot4) \\ z &= [9\cdot69434]\, r \sin (v + 146\ 48\cdot7) \end{aligned}$$

T is left to be determined from the observations. The value finally adopted is 1818 February 6·30 G.M.T.

As rectangular co-ordinates of the Sun were not given in the Almanacs at that epoch, it is convenient to give the values that I have computed from the data of the *Berliner Jahrbuch*. The observations are so rough that a determination from modern tables is not called for.

Pons gave his time of observation as 7^{h} Marseilles time; making a preliminary estimate of the light-time as $6^{m}\cdot3$, I made the computations for $6^{h}\ 34^{m}\cdot3$ G.M.T.

G.M.T.	log X.	log Y.	log Z.
1818 Feb. 23·274	9·95156	9·59047*n*	9·22806*n*
24·274	9·95521	9·57417*n*	9·21176*n*
25·274	9·95869	9·55709*n*	9·19468*n*
26·274	9·96200	9·53915*n*	9·17673*n*
27·274	9·96517	9·52032*n*	9·15792*n*

Herr Oberhofmeister Freiherr von Zach published in *Zeitschrift für Astronomie*, Jan. und Feb. 1818, p. 150, the following extract from a letter received by him from J. L. Pons, dated 1818 February 28. I think it well to reproduce this *in extenso*, as it is the sole source of information concerning the apparition of our comet in 1818.

"Am 23 Febr. entdekte Herr *Pons*, Abends um 7 Uhr abermals einen Cometen auf der Brust des Wallfisches in 15° 15′ südlicher

Abweichung; er beschreibt ihn also: 'Dieser Comet ist beinahe eben so klein, als der vorhergehende, man sieht ihn nicht mit blosen Augen, und er verträgt keine Beleuchtung. Er hat weder Schweif noch Bart. Sein Mittelpunct ist etwas heller, der Nebel wenig ausgedehnt. Am 24 Febr. sah ich ihn nur auf wenige Augenblicke, wegen des schlechten Wetters, ich habe jedoch so viel erkannt, dass er eine ziemlich schnelle Bewegung in gerader Aufsteigung hat, etwa 7 Minuten in Zeit gegen Osten, und 40 Min. in Abweichung gegen Süden. Den 26 Febr. gieng er ungefehr 3 Minuten nach einem Stern 4 Grösse durchs Feld des parallactischen Fernrohrs. Dieser Stern ist der südlichste von den vier Sternen, welche auf der Brust des Wallfisches ein Viereck bilden. Den 27 Febr. war der Comet im Felde des Fernrohrs mit einem Stern unter den Pfoten des Wallfisches, etwa 50 Min. östlicher. Es ist ungefehr 14 Tage, dass ich einen schwachen Nebelfleck etliche Grade unter dem Stern *B* im Wallfisch bemerkt hatte, welchen mir böses Wetter wieder zu sehen verhindert hatte, ich vermuthe beinahe, dass diess der Comet gewesen sei. Vermög seines Laufes, wird er sich nun bald unter dem Horizonte verlieren.'

Aus allen diesen Datis erhellet, dass der Comet am 26 Febr. mit dem Stern σ, den 27 Febr. mit τ im Wallfisch verglichen worden ist. Hieraus lassen sich zwar keine sichere Stellungen herleiten, indessen gebe ich hier folgende Conjecturen, aus welchen man wenigstens wird erkennen können, ob es derselbe Comet ist, im Falle er irgendwo beobachtet worden wäre.

1818 Febr.	23, 7 Uhr M.,	A. R. Comet	31	15,	Abw.	15	15 südl.
	24 ,,	,,	33	0		16	55 südl.
	26 ,,	,,	36	30		17	35 südl.
	27 ,,	,,	36	18		18	15 südl."

With regard to the estimates of position by von Zach, it would appear that he intended to use Pons's statement of daily motion + 1° 45′, south 40′. There are, however, some slips: this supposition would give R.A. on February 27, 38° 15′, and Decl. on February 24, 26, 27, 15° 55′, 17° 15′, 17° 55′ respectively. It is probable that Pons gave the Decl. on February 23 simply as 15° south, and that von Zach inserted the 15′ as correction for index error and refraction (see his previous communication on p. 149 of the *Zeitschrift*, where he states that the index reading is 11½′ too far north).

The star that von Zach erroneously calls τ Ceti is Flamsteed's 90 Ceti,* now known as τ^1 Eridani. The following are the positions of stars for 1818·0 :—

Star.	Mag.	R.A. 1818·0. h m s	S. Decl. 1818·0. ° ′
β Ceti	2·2	0 34 27·3	18 59·2
σ Ceti	4·8	2 23 27·88	16 2·9
Lalande 4976	6·9	2 31 27·1	17 5·8
τ^1 Eridani	4·6	2 36 36·69	19 20·9

* Erroneously designated 80 Ceti in Boss P.G.C.

I insert Lalande 4976, since it seems to me possible that this was the star used by Pons on February 27: it was nearer to the comet in declination than τ^1 Eridani; also the comet was to the *east* of it, whereas it was to the *west* of τ^1.

We deduce $2^h\ 26^m\ 27^s{\cdot}88$ or $36^\circ\ 36'{\cdot}97$ for the observed R.A. of the comet on February 26. I take it that the 50′ on February 27 is to be reckoned in great circle, since it was apparently estimated by distance in the field of view, not by transits. We deduce that the observed R.A. on February 27 was $38^\circ\ 44'{\cdot}29$ if Lal. 4976 was the star used, $38^\circ\ 16'{\cdot}53$ if it was τ^1 Eridani. I have constructed the following ephemerides on two assumptions of the date of perihelion.

T 1818 Feb. 6·921 G.M.T.

1818 G.M.T.	R.A.	S. Decl.	log r.	log Δ.
	° ′	° ′		
Feb. 23·2724	31 24·96	13 46·89	9·90752	9·84809
24·2724	32 59·93	14 25·73	9·91151	9·84498
25·2724	34 35·22	15 3·64	9·91559	9·84217
26·2724	36 12·04	15 42·28	9·91985	9·83944
27·2724	37 49·54	16 20·36	9·92422	9·83697

T 1818 Feb. 5·921 G.M.T.

Feb. 23·2724	32 18·73	15 13·44	9·91151	9·83744
24·2724	33 54·48	15 52·62	9·91559	9·83464
25·2724	35 31·50	16 32·63	9·91985	9·83191
26·2724	37 9·62	17 11·91	9·92422	9·82944
27·2724	38 48·00	17 50·53	9·92867	9·82722

By interpolation we find that T = February 5·903 from the declination on February 23, February 6·488 from the R.A. on February 26, February 5·984 from the R.A. on February 27 using Lalande, February 6·459 from that R.A. using τ^1. I assign weights 2, 4, 1, 2 to these. The highest weight is given to the R.A. from σ Ceti, since there is no doubt about the identity of the star in this case. The weighted mean is February 6·296, which is unlikely to be more than 0·25 days in error. The average daily motion was $1^\circ\ 36'{\cdot}9$, south $39'{\cdot}0$; Pons's estimate of $1^\circ\ 45'$, south $40'$, was tolerably good. The computers of the orbit gave great weight to this estimate, which explains the fact that fair orbits were deduced from such unsatisfactory material.

In view of Pons's statement about an object seen a fortnight earlier, I calculated the following positions (from slightly different elements).

	R.A.	S. Decl.	log Δ.
	h m s	° ′	
1818 Feb. 14·2	1 13 52	8 23	9·8802
15·2	1 19 44	8 58	9·8761

These show that the comet was fully 10° to the north of β Ceti,

which I assume to be the object meant by *B* Ceti. Hence the object seen a fortnight earlier was not this comet.

It is of interest to compare the perturbations calculated for ω, Ω, and i from 1873 to 1928 with the changes shown by the orbits given above : they are as follows :—

	Calculated.	Observed.
ω	− 9′·5	−13′·7
Ω	−30·5	−26·2
i	+ 5·6	+ 6·8

It will be noticed that the orbit given for 1928 is not definitive, and that the change of the period from $28^{y}\cdot655$ to $27^{y}\cdot89$ would have some effect on the other elements. The agreement is as close as we could expect.

I have not applied perturbations by the Earth or Venus ; the closest approach to the Earth was at the 1873 perihelion when the least distance was 0·219 (very nearly the least possible value). In 1818 it was about 0·6, in 1928 0·47 ; the distances in 1845 and 1901 were greater. Venus can make close approaches to the comet, the distance between the orbits near the comet's descending node being only about 0·04. For such an approach the longitude of Venus would have to be 69°·8 eight days before the comet's perihelion passage, or 83° at the perihelion passage. The actual longitudes of Venus at the different perihelia were 1818 300°, 1845 305°, 1873 201°, 1901 49°, 1928 301°. The approach to the earth in 1873 appears to be the only one that would be likely to give appreciable perturbations.

There are some fairly close commensurabilities between the comet's period and those of the large planets. Thus seven periods of Jupiter are equal to 83·03 years, which happens to be exactly the interval between the comet's perihelion passages in 1845 and 1928. Also six periods of the comet are very close to Neptune's period, and three periods to that of Uranus. In view of the smallness of the perturbations due to these planets, it will suffice for most purposes to take them as repeating themselves after these periods.

The period of the comet is fairly close to that of Saturn, so the conditions change rather slowly from revolution to revolution. The comet is at Saturn's distance from the Sun $3\frac{1}{2}$ years before and after perihelion. The following table shows the values of $L_1 - u$ and B_1 in the different revolutions, and the distance of the comet from Saturn.

Perihelion.	Before Perihelion.			After Perihelion.		
	$L_1 - u$.	B_1.	Dist.	$L_1 - u$.	B_1.	Dist.
	°	°		°	°	
1818	..	..	..	140·7	−23·5	18
1845	350·0	−13·2	3	116·8	−29·2	17
1873	337·2	− 6·2	4	98·0	−30·7	15
1901	318·6	+ 4·8	7	71·4	−28·0	12
1928	298·8	+15·5	10			

The perturbations before perihelion reached a maximum about 1845 ; those after perihelion will reach a maximum about three revolutions after 1928.

We can make a fair estimate of the date of perihelion in 1956 by using the above relations. The following are the estimated perturbations :—

	M_0.	n.	n (second sum.).
Jupiter	+1764″	+″·3225	−2678″
Saturn	− 190	0	+ 700
Uranus	− 21	+·0021	+ 2
Neptune	− 5	−·0015	− 13
Sum.	+1548	+·3231	−1989

I assume 27ʸ·89 for the period in 1928, which makes n = 127″·221. Then n_{1928} × interval in days − 441″ = 1296000″.

This gives 10,190 days and makes the estimated perihelion passage 1956 September 29, the period then being 27ʸ·82. It will be a less favourable return than the recent one, but the comet should be observable without difficulty. The longitude of Venus will be about 69°, so there is likely to be a much closer approach to that planet than at any time since 1818.

In conclusion I quote Toscanelli's observations of comet 1457 I, as given by L. Schulhof in *B.A.*, **3**, 134 ; there is good reason for suspecting that this is the same comet ; if so, 13 revolutions were completed between 1457 and 1818, giving a mean period of 27·77 years.

1457 6^h Paris M.T.	Longitude.	Latitude.
Jan. 23	358° 13′·5	− 7° 9′·0
24	359 33·0	− 7 33·0
25	0 26·1	− 8 46·3
26	1 36·0	− 9 39·9
27	2 30·0	−10 29·7

It is obvious from internal evidence that these are liable to errors of at least 15′. Schulhof showed that they could be satisfied by his elements of this comet within quantities of about the same order of magnitude. Celoria deduced the following orbit from these observations before the identity was suggested, so that they are quite independent :—

T 1457 Jan. 18·0, ω 194°·9, Ω 249°·7, i 13°·3, log q 9·847.

The only discordance is in the inclination, and the comet was too near the ecliptic for this to be very accurately deduced from such rough observations.

Emission Lines on the Absorption Bands of H *and* K.

By J. Evershed, F.R.S.

In *Monthly Notices*, **87**, 350, I recorded the presence of an emission line in the solar spectrum near K, having the approximate wave-length 3934·80 I.A., or 3934·94 on Rowland's scale. I now find that this line was observed thirty years ago by L. E. Jewell, who gives the wave-length 3934·954. Jewell at first thought the line might be a ghost of the reversal of K, but finally concluded that "there is no question whatever as to the chromospheric origin of the line" (*Astrophysical Journal*, **8**, 119, 1898).

The bright line is only found close to the limb of the Sun, and cannot be traced over the disc more than about one-twentieth of the solar radius. In some recent photographs obtained under the best atmospheric conditions, with the slit of the spectrograph so nearly tangent to the limb that the overlap would not exceed 5″, three other bright lines have been detected. In these plates the 3934 line stands out very clearly, whilst a less conspicuous emission line is found on the other side of K nearly twice the distance from its centre. In addition, in the H region there appears to be a strong emission line bordering the iron line λ3969·270 on the red side, and a very faint emission line on the other side of H, and further from its centre. This is difficult to detect on the brighter background except on plates that have been given exactly the right exposure, and which have not overlapped the limb too much. It is sufficiently distinct on one or two of my plates to allow of fairly accurate measurement.

The approximate positions of the four emission lines on my plates, and of the centres of H and K, as given in the recently published Revision of Rowland's "Preliminary Table," are as follows :—

Line.	λ I.A.	Intensity.
Emission 1	3931·54	0
K	3933·684	1000
Emission 2	3934·80	2
Emission 3	3967·04	0
H	3968·494	700
Emission 4	3969·40	3

The wave-lengths of the calcium lines presumably refer to the centres of H_2 and K_2 as observed at the centre of the Sun's disc, whilst the wave-lengths of the emission lines are derived from solar iron lines in the limb spectra. These values are subject to a small positive correction for limb-effect, which would, however, not amount to more than two or three thousandths of an angstrom.

According to the Revised Table of Rowland, an absorption line occurs at 3931·590, intensity 1, but marked as doubtful. This is almost coincident with the emission line No. 1, but does not exist in any of my spectra, either of the limb or of the centre of the disc. Also near

H a line is given at 3967·057, attributed to Ce^+, intensity 0, practically coincident with emission line No. 3. This line also I fail to find in any of my spectra. Lastly, a line at 3969·407, coinciding with the emission line No. 4, does appear as a very faint absorption line in one of my spectra of the centre of the disc.

The H_2 and K_2 lines of Ca^+ are themselves emission lines near the limb, and it is curious that each should be accompanied by an emission line on either side of it, the one on the more refrangible side in each case being further from the centre of the *Ca* line, and the other closer in on the deeper shading of the *Ca* band. But the relative distances are not the same in the two cases, as will be seen from the measures of wave-length.

It appears that the combined effect of the general absorption of the continuous spectrum near the limb and the dark bands forming the wings of H and K determine the emissive character of the lines. Yet absorption lines are also found superposed on the bands, even on the darker parts of the shading. It is remarkable that the emission does not extend into the upper chromosphere, and it cannot be traced in my plates beyond the limits of the photosphere. Evidently it originates above the densest layer of calcium gas, and yet at a low level in the reversing layer.

These lines should show strongly in spectra of the narrow cusp taken just before or after total eclipse, but high dispersion would be required to separate them from H and K.

Ewhurst:
1929 *April* 9.

The Determination of Stellar Distances from Intensities of the Detached Calcium Line K. By Otto Struve, Ph.D.

I. *Outline of the Paper.*

In an earlier paper (1) I have shown that the observed intensities of the detached Ca^+ line K in stars of type earlier than B5 are strongly correlated with the distances of the stars. This result has since been confirmed in a number of ways. In Section II. of this paper it is shown that the various methods that have been applied lead to fully consistent results, and that a definite empirical relationship between Ca^+ intensity and distance can be established.

In Section III. we invert the process and determine, from the empirical curve, the distances of individual stars. This is a logical consequence of the distance-intensity relationship, and the resulting values should serve as a final check upon the whole work. The list, Table V., includes nearly all stars for which good one-prism slit-spectrograms were available at the Yerkes Observatory, and is consequently limited to the brighter apparent magnitudes. This limitation enabled me to supply for all stars several other spectroscopic characteristics of interest, such as width and intensity of the ordinary stellar absorption lines. At the

same time these brighter stars have the best-determined Ca^+ intensities, the essential correctness of which can easily be checked at other observatories. These intensities should be correct within the limits given by the probable error of $\pm$ 1 arbitrary unit.

The proposed method of determining spectroscopic parallaxes differs from other methods in that it is the distance itself that is directly determined from the intensities, and not the absolute magnitude or the parallactic angle. Furthermore, it involves no assumption concerning the masses of the stars. On the other hand, we do not yet know the dispersion of the actual distances from the mean curve, and it is not yet apparent whether the new procedure will, in practice, give better results than the earlier methods.

Other spectroscopic parallax methods determine, in the first place, the absolute magnitude, and the distance is a by-product supplied by the apparent magnitudes. In the proposed method the absolute magnitudes are a by-product computed from the distances and the apparent brightnesses. They are given in column 7 of the main table.

The validity of a new method for the determination of spectroscopic parallaxes is usually tested by means of various statistical criteria of distance. It follows from Section II. that the method under discussion meets these requirements. However, we shall see that a more severe test can be applied. An inspection of the absolute magnitudes in Table V. reveals a considerable dispersion within each spectral subdivision. This is not contrary to expectation ; indeed, we know from the work of R. E. Wilson (2) that the absolute magnitudes of the O-stars have a large dispersion. Recent investigations by R. O. Redman (3) and by J. J. Nassau (4) have shown that the dispersion is also appreciable for the later spectral types of the dwarf series. Furthermore, for the B-stars a considerable dispersion is indicated in the statistical work of R. E. Wilson (5), of B. P. Gerasimovič (6), and in my own statistical results (7). Finally, early-type stars are known to show a large dispersion in mass as determined from eclipsing binaries observed spectroscopically. Since there seems to be very little scattering from the mass-luminosity curve, it follows that the dispersion in absolute magnitude must be large.

An ultimate test of the method would be provided if it could be shown that the absolute magnitudes computed from the Ca^+ distances have a real physical meaning. This has been done in four partly independent ways. In Section IV. I compare my absolute magnitudes with those of Adams and Joy. The latter are based almost exclusively upon an accurate spectral classification. They do, however, admit of a small dispersion in M within each spectral subdivision by the introduction of the "s" and "n" characteristics. It appears that (in general) within each subdivision the bright absolute magnitudes of Adams and Joy correspond to the bright absolute magnitudes as derived from the detached Ca^+ lines.

Section V. contains a similar comparison with the absolute magnitudes of D. L. Edwards determined at the Norman Lockyer Observatory. In his work, as in that of Adams and Joy, spectral classification was

the chief factor, so that comparatively little dispersion was allowed within each subdivision. Nevertheless, a satisfactory correlation is obtained between his values and mine.

In Section VI. I discuss the correlation between absolute magnitude and line-width. This is, of course, related to the " n " and " s " effect of Adams and Joy. The correlation turns out to be fairly strong. Thus in subdivision B3 the stars having the narrowest lines are by nearly 2 magnitudes more luminous than the stars with particularly wide lines.

An attempt was also made to find a spectral characteristic depending upon absolute magnitude. This attempt was only partially successful. The line-spectra of absolutely bright stars and of faint stars show very little difference, as was indeed found earlier by Adams and Joy. The difference in intensity of the two helium lines $\lambda 4472$ and $\lambda 4388$ was shown to be correlated with absolute magnitude in subdivisions earlier than B3. A discussion of this property is given in Section VII.

In the last section I compare my empirical results with the demands of theory. A. S. Eddington's hypothesis of interstellar matter, in conjunction with the atomic absorption coefficient as given by the classical electron theory, leads to a determination of the theoretical contour of the detached line. A comparison with the observations established the density of ionized calcium atoms in space. The value thus found is 5×10^{-30} gr. per cm.^{3}. The theoretical distance-intensity relationship depends markedly upon the calibration of my arbitrary scale of central intensities. However, only a crude calibration is available. The empirical curve differs in shape from the theoretical curve, but the disagreement can be accounted for by errors in the calibration.

It should be noted that I do not consider the distances given in this paper to be sufficiently well established to supersede those determined by other methods. They are given here merely as a basis for comparison, and are intended in the first place as a check on our assumption that intensity is a function primarily of distance. While the results seem to justify this assumption, it is by no means certain that the method used here will prove to be of any great practical value in the determination of stellar parallaxes. If it should turn out that the Ca^+ distances are not identical with the actual distances, within the errors of observation, the differences will doubtless serve to elucidate any factors other than distance that may affect the intensities of the detached lines.

II. *The Distance-Intensity Relationship.*

In this section we shall briefly summarize all the evidence derived previously in favour of the idea that the intensity of detached K increases with distance. This will enable us to construct a new empirical curve $i = f(D)$ (shown in fig. 1) for the derivation of stellar distances from the observed intensities. The latter are expressed in arbitrary units, defined in such a way that the central intensity of K in Vega is 10. A crude calibration has shown that one unit of my

scale corresponds roughly to a step of $0^{m}\cdot1$ in the differences in intensity between continuous spectrum and centre of line.

(*a*) The original curve $i = f(D)$ was derived from a number of physically connected groups of stars for which the distances had been obtained by other methods (8). These are enumerated, with a few additions, in Table I., and are shown by black dots in fig. 1.

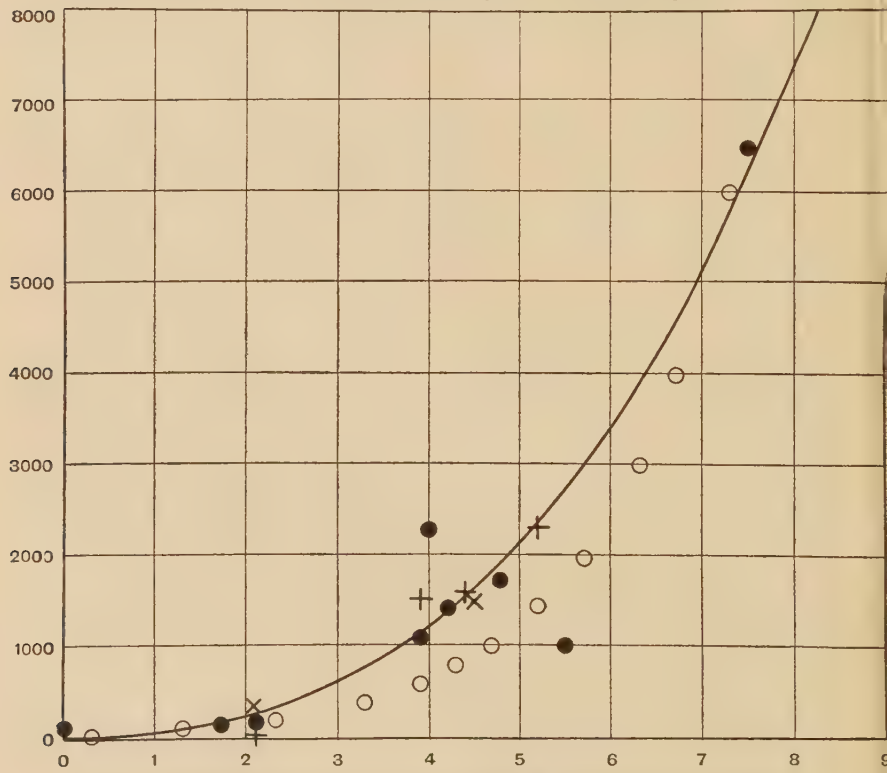

FIG. 1.—Relationship between Ca^{+} intensity in arbitrary units (abscissæ) and distance in parsecs (ordinates).

TABLE I.

Group.	Mean *i*.	D in Parsecs.
1. Stars of type B5, app. mag. 5 (66 Eridani)	0·0	100
2. Eddington's moving cluster in Perseus	1·7	150
3. Orion nebula group	2·1	180
4. Cluster in Cygnus	3·9	1100
5. Cluster in Perseus (*h* and χ)	4·0	2300
6. Oe5 stars	4·2	1450
7. Oe stars	4·8	1730
8. Three galactic novæ	5·5	1000
9. P Cygni stars	7·5	6500

(*b*) The mean intensities have been shown in a former paper (9) to increase linearly with apparent magnitude. The data in table ix. of that paper are satisfactorily represented by the equation :

$$i = 0{\cdot}35m + 0{\cdot}82 \quad . \quad . \quad . \quad . \quad (1)$$

In order to find the relationship between i and absolute magnitude, M, we introduce :

$$m = \mathrm{M} - 5 + 5\log_{10}\mathrm{D} \quad . \quad . \quad . \quad (2)$$

It has been shown by R. E. Wilson (10), by Charlier (11), by Gerasimovič (12), and by myself that M is itself a function of apparent magnitude m. Since the majority of our stars belong to type B3, I shall use the value of M as determined by Gerasimovič for this subdivision. I have adopted a linear relationship between M and m in the form :

$$\mathrm{M} = 0{\cdot}48m - 4{\cdot}20 \quad . \quad . \quad . \quad . \quad (3)$$

Combining (1), (2), and (3) we get

$$i = 3{\cdot}36\log\mathrm{D} - 5{\cdot}38 \quad . \quad . \quad . \quad (4)$$

From this equation we compute the following table. The respective values are represented in fig. 1 by open circles.

D.	*i*.	D.	*i*.	D.	*i*.
50	0·3	600	3·9	3000	6·3
100	1·3	800	4·3	4000	6·7
200	2·3	1000	4·7	6000	7·3
400	3·3	1500	5·2	10,000	8·0

(*c*) Several additional points for fig. 1 were obtained from the Ca^+ residual radial velocities. I have shown elsewhere (13) that these velocities may be explained by J. H. Oort's galactic rotation :

$$\mathrm{V} = \mathrm{K} + \mathrm{D.A.}\sin 2(l - 325^\circ).$$

For the Ca^+ lines K = 0. All radial velocities together give $\mathrm{D_0A} = +5{\cdot}3 \pm 1{\cdot}0$ km./sec. Individual groups of stars give, according to their distances, different values, $\mathrm{D}_n\mathrm{A}$, and the ratio $\mathrm{D}_n/\mathrm{D}_0$ can be determined. The results for several such groups are summarized in Table II. The third column contains the D_n under the assumption that $\mathrm{D}_0 = 800$ parsecs. These values are marked by crosses (+) in fig. 1.

Table II.

Group.	$\mathrm{D}_n/\mathrm{D}_0$.	D_n.	Mean *i*.
Eight faint stars in Perseus	2·9	2300	5·2
Cygnus cluster	1·9	1500	3·9
Orion group	0·2	160	2·1
30 τ Can. Maj.	2·0	1600	4·4

(*d*) Greaves, Davidson, and Martin (14), and also Gerasimovič (15), have recently confirmed my conclusion (16) that stars with strong detached Ca^+ lines are usually yellower than stars with faint lines. Lists of "yellow" and of "white" B-stars have been published by E. Hertzsprung (17), who noted that the "yellow" stars are strongly concentrated towards the galactic plane, while no such concentration is observed in the "white" stars. Consequently, the "yellow" stars must be much farther away than the "white" ones. For the stars of type O to B2 the following mean Ca^+ intensities are found :—

14 "yellow" stars ; mean $i = 4{\cdot}5 \pm 0{\cdot}4$.
4 "white" stars ; mean $i = 2{\cdot}1 \pm 0{\cdot}3$.

It is hardly possible to derive a significant value of the relative distances of the "yellow" and "white" stars, but a rough estimate can be made in the following way. Assuming that all of Hertzsprung's "yellow" stars form a homogeneous group, we can estimate their mean distance from the "galactic dip." The mean galactic latitude of his 41 "yellow" stars is $-1°{\cdot}3$. Assuming with Gerasimovič and Luyten (18) that the elevation of the Sun over the galactic plane is 33 parsecs, we find for the "yellow" stars a distance of about 1500 parsecs. The distance of the "white" stars may be estimated at about 200 parsecs. These values are shown in fig. 1 by the sign ×.

(*e*) The curve in fig. 1 can be shown to be consistent with the established fact (19) that the Ca^+ intensity is greatest in the earliest types. The mean intensities for the various spectral subdivisions here considered, and the corresponding absolute magnitudes M, are shown in Table III.

TABLE III.

Spectrum	O	Bo	B1	B2	B3
Intensity	4·4	3·7	3·1	3·6	3·0
Smoothed intensity	4·4	3·7	3·3	3·1	3·0
Absolute magnitude	−4·0	−3·1	−2·5	−1·8	−1·2

Differentiating equation (2) and considering m as constant, we get

$$\Delta D = -\frac{D}{2{\cdot}2}\Delta M \quad . \quad . \quad . \quad . \quad (5)$$

The curve combining intensity and distance agrees closely with the empirical equation

$$i = c\sqrt{D} \quad . \quad . \quad . \quad . \quad (6)$$

where $c = 0{\cdot}107$. Consequently

$$D = \frac{i^2}{c^2}; \qquad \Delta D = \frac{2i}{c^2}\Delta i \quad . \quad . \quad . \quad (7)$$

Substituting these in (5) we get

$$\Delta i = -0{\cdot}23\,.\,i\,.\,\Delta M \quad . \quad . \quad . \quad (8)$$

From this expression we compute the expected values of Δi, using for i the mean intensity corresponding to every given interval, Δ (spectrum). The results are summarized in Table IV.

TABLE IV.

Δ (Spectrum).	Δi (Observed).	Δi (Computed).
O–B3	+1·4	+2·4
Bo–B3	+0·7	+1·5
B1–B3	+0·1	+0·9
B2–B3	+0·6	+0·5

The computed values are somewhat larger than the observed ones, but they are of the same sign and order of magnitude. The remaining discrepancy is probably due to our simplifying assumption, m = const.

(*f*) Our relationship $i = f(D)$ suggests that the lines must be strongest in the vicinity of Shapley's centre of the galactic system—in longitude 325°. I have shown that this is actually the case (20); the lines are strongest in longitude 338° ± 18°, and weakest in the opposite point of the galactic circle.

(*g*) Proper motion is usually considered as a good indicator of distance. Accordingly we expect from our distance-intensity relationship a definite correlation between proper motion and intensity. That such a correlation is present I have previously shown (21) on the basis of the Yerkes Observatory spectrograms:

P.M.	No.	*i*.	P.M.	No.	*i*.
″ ″ 0·000–0·009	53	3·2	″ ″ 0·040–0·049	11	2·2
0·010–0·019	43	3·1	0·050–0·079	3	1·3
0·020–0·029	22	3·0	0·080–	2	0·5
0·030–0·039	18	1·9			

(*h*) On the basis of the same material I have shown that the larger trigonometric parallaxes from Schlesinger's catalogue are associated with fainter lines than the smaller parallaxes:

	Mean π.	No.	Mean *i*.
Negative π	−0·011	14	3·1
Positive π	+0·011	15	2·7

(*i*) It has been shown that the components of physical double stars have identical Ca^+ intensities (35). Consequently G. Shajn's (36) test for spectroscopic parallaxes, $\Delta M = \Delta m$, is fulfilled.

All the evidence collected goes to show that the relationship between intensity and distance is real. We have therefore adopted for all further computations the functional relationship shown by the full line in fig. 1. This curve has been drawn to represent chiefly the values derived under (*a*), which are considered the most reliable.

III. *Observational Material.*

Table V. contains the results of observations. The various columns give: the name of the star; its position in α and δ; the apparent photographic magnitude, m; the observed intensity of detached K, i; the resulting distance, D; the absolute magnitude computed from $M = m + 5 - 5 \log D$; the central intensities of the stellar lines, *He* 4388, *He* 4472, and H_γ (on a scale similar to that employed for K); and the width of *He* 4472 on an arbitrary scale.

Calibrations of the scales of intensity and of line-width are given below:

Calibration of Intensities.

i in arbitrary units	1	2	3	4	5	6	7	8	9	10
i in stellar magnitudes	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
i in percentage absorption	9	17	24	31	37	42	47	52	56	60

Calibration of Line-Widths.

Width in arbitrary units	1	2	3	4	5	6	7	8	9	10
Width in angstroms	1·2	1·4	1·7	1·9	2·1	2·3	2·5	2·8	3·0	3·2

It should be noted that both calibrations, especially that of the intensities, are crude. Reliable microphotometric determinations of central intensities can be obtained for fairly strong lines only. The absolute intensities here given cannot claim to give more than the order of magnitude of the quantities involved.

The stars in Table V. are arranged according to Harvard spectral type.

TABLE V.

Star.	α.	δ.	m.	i.	D.	M.	Line 4388.	Line 4472.	Line H_γ.	Width.
Type O.										
ξ Per.	3 52·5	+35 30	4·0	3·0	650	− 5·0	1	5	10	7
λ Ori. br.	5 29·6	+ 9 52	3·7	5·1	2300	− 8·1	1	10	10	4
ι Ori.	30·5	− 5 59	2·9	1·5	150	− 3·0	2	7	10	4
S Mon.	6 35·5	+ 9 59	4·7	6·1	3600	− 8·1	2	4	5	6
A Cyg.	21 14·8	+43 31	5·1	4·4	1600	− 5·9	1	1	4	10
13 Hev. Cep.	35·9	+57 2	5·6	5·5	2700	− 6·2	3	4	8	5
19 Cep.	22 2·1	+61 48	5·2	6·5	4300	− 8·0	5	15	7	5
λ Cep.	8·1	+58 56	5·2	6·1	3600	− 7·6	0	4	6	10
10 Lac.	34·8	+38 32	4·9	3·7	1000	− 5·1	5	10	8	2
Type Bo.										
$\varkappa$ Cas.	0 27·3	+62 23	4·0	2·7	500	− 4·5	7	10	9	4
9 α Cam.	4 44·1	+66 10	4·1	8·4	8300	−10·5	1	10	6	3
+34° 980	5 9·7	+34 12	5·6	2·7	500	− 2·9	7	10	20	1

TABLE V.—*continued.*

Star.	α.	δ.	*m.*	*i.*	D.	M.	Line 4388.	Line 4472.	Line H_γ.	Width.
Type Bo.—*continued.*										
δ Ori.	5 26·9	− 0 22	2·0	1·7	200	− 4·5	3	9	10	6
ϕ^1 Ori.	29·3	+ 9 25	4·3	3·2	700	− 4·9	8	10	12	2
ε Ori.	31·1	− 1 16	1·5	1·9	200	− 5·0	4	7	10	5
σ Ori.	33·7	− 2 39	3·5	1·3	100	− 1·5	7	10	15	4
ζ Ori.	35·7	− 2 0	1·8	2·1	300	− 5·6	1	4	5	7
κ Ori.	43·0	− 9 42	2·0	2·8	550	− 6·7	5	10	12	4
ϱ Leo.	10 27·5	+ 9 49	3·6	3·8	1100	− 6·6	7	5	12	3
δ Sco.	15 54·4	−22 20	2·3	1·0	100	− 2·8	8	10	12	9
τ Sco.	16 29·7	−28 1	2·7	0·8	50	− 0·8	7	8	10	3
ζ Oph.	31·7	−10 22	2·5	1·3	100	− 2·5	0	2	3	10
2 Vul.	19 13·5	+22 51	5·2	2·3	350	− 2·5	3	4	10	10
κ Aql.	31·5	− 7 15	4·8	2·2	300	− 2·6	2	5	7	9
f^1 Cyg.	20 56·4	+47 8	4·6	0·6	50	+ 1·1	1	1	3	5
14 Cep.	21 58·7	+57 31	5·3	4·1	1400	− 6·4	0	6	3	5
Type B1.										
ε Per.	3 51·1	+39 43	2·7	1·7	200	− 3·8	5	7	10	7
1 Cam.	4 24·1	+53 42	5·2	5·5	2700	− 7·0	1	2	4	10
λ Lep.	5 15·0	−13 17	4·1	2·0	250	− 2·9	9	10	13	2
η Ori.	19·4	− 2 29	3·2	1·5	150	− 2·7	1	3	5	5
χ Aur.	26·2	+32 7	4·7	4·7	1900	− 6·7	8	10	20	1
−6° 1234	30·1	− 6 5	4·5	2·2	300	− 2·9	10	8	12	5
β CMa.	6 18·3	−17 54	1·8	1·0	100	− 3·2	8	10	15	3
15 CMa.	49·2	−20 6	4·4	2·1	300	− 3·0	10	10	15	4
ε CMa.	54·7	−28 50	1·4	0·6	50	− 2·1	10	12	11	2
β Sco.	15 59·6	−19 32	2·7	1·1	100	− 2·3	4	7	9	5
σ Sco.	16 15·1	−25 21	2·9	1·9	200	− 3·6	8	10	12	2
β Cep.	21 27·4	+70 7	3·1	0·5	50	− 0·4	12	10	10	1
π Aqr.	22 20·2	+ 0 52	4·4	2·8	550	− 4·3	1	4	em	10
1 Cas.	23 2·4	+58 53	4·7	4·2	1400	− 6·0	8	10	15	3
Type B2.										
o Cas.	0 39·2	+47 44	4·5	1·0	100	− 0·5	2	3	20	10
δ Cet.	2 34·4	− 0 6	3·8	0·8	50	+ 0·3	10	8	8	1
o Per.	3 36·0	+33 39	4·8	1·9	200	− 1·7	8	6	10	4
ν Eri.	4 31·3	− 3 33	3·9	2·8	550	− 4·8	12	9	13	3
γ Ori.	5 19·8	+ 6 16	1·5	2·1	300	− 5·9	10	10	11	4
ψ Ori.	21·6	+ 3 0	4·5	1·1	100	− 0·5	8	6	10	8

TABLE V.—*continued.*

Star.	α.	δ.	m.	i.	D.	M.	Line 4388.	Line 4472.	Line H_γ.	Width.
				Type B2.—*continued.*						
−1° 935	5 27·7	− 1 40	5·1	3·5	900	− 4·7	8	10	15	1
57 Ori.	49·1	+19 44	5·7	5·1	2300	− 6·1	6	8	12	7
139 Tau.	51·8	+25 57	4·7	5·8	3100	− 7·8	5	10	12	8
χ^2 Ori.	58·0	+20 8	4·5	8·7	9200	−10·3	9	9	12	2
ν Ori.	6 1·9	+14 47	4·2	2·5	450	− 4·1	7	8	12	4
−11° 1460	16·8	−11 44	5·3	4·2	1400	− 5·5	1	4	em	7
α Vir.	13 19·9	−10 38	1·0	0·2	30	− 1·4	2	2	5	10
π Sco.	15 52·8	−25 50	2·8	0·9	100	− 2·2	6	6	10	7
ω^1 Sco.	16 1·0	−20 24	3·9	0·8	50	+ 0·4	5	7	9	5
55 Cyg.	20 45·5	+45 45	4·7	6·7	4700	− 8·7	8	10	20	3
+59° 2334	21 9·3	+59 35	5·5	4·6	1800	− 5·8	6	10	12	1
9 Cep.	35·2	+61 38	4·7	7·0	5200	− 8·9	7	15	10	3
12 Lac.	22 37·0	+39 43	5·0	4·0	1300	− 5·6	9	10	10	1
σ Cas.	23 53·9	+55 12	4·7	2·1	300	− 2·7	5	7	10	7
				Type B3.						
ζ Cas.	0 31·4	+53 21	3·6	3·8	1100	− 6·6	10	10	17	3
π And.	31·5	+33 10	4·3	1·3	100	− 0·7	3	4	8	6
ξ Cas.	36·5	+49 58	4·7	2·5	450	− 3·6	2	3	7	8
ϵ Cas.	1 47·2	+63 11	3·3	3·8	1100	− 6·9	6	10	15	3
35 Ari.	2 37·6	+27 17	4·4	1·0	100	− 0·6	4	10	20	6
1 Hev. Cam.	3 11·2	+65 17	4·6	1·3	100	− 0·4	0	1	em	10
29 Per.	11·5	+49 51	5·1	2·0	250	− 1·9	3	7	10	6
31 Per.	12·0	+49 43	4·9	1·4	150	− 1·0	0	2	6	10
τ Ari.	15·5	+20 47	5·0	5·5	2700	− 7·2	7	8	11	2
+48° 899	16·1	+48 51	5·1	1·1	100	+ 0·1	2	4	12	10
29 Tau.	40·3	+ 5 44	5·2	1·8	200	− 1·3	6	8	15	8
+34° 768	50·0	+34 47	5·3	2·0	250	− 1·7	3	8	10	9
c Per.	4 1·4	+47 27	3·9	1·0	100	− 1·1	2	3	em	10
−16° 796	4·7	−16 39	5·3	3·0	650	− 3·8	2	10	15	4
+83° 104	5·0	+83 34	5·2	1·5	150	− 0·7	0	2	20	6
μ Tau.	10·1	+ 8 39	4·2	1·1	100	− 0·8	8	8	12	3
+50° 973	12·6	+50 41	5·4	2·0	250	− 1·6	4	6	13	4
d Per.	14·3	+46 16	4·7	4·2	1400	− 6·0	9	10	13	2
π^4 Ori.	45·9	+ 5 26	3·6	2·3	350	− 4·1	8	10	17	3
π^5 Ori.	49·0	+ 2 17	3·7	2·5	450	− 4·6	6	7	10	5
11 Cam.	57·4	+58 50	5·1	2·0	250	1·9	2	5	em	5
η Aur.	59·5	+41 6	3·1	1·2	100	1·9	4	8	20	8

TABLE V.—*continued.*

Star.	α.	δ.	m.	i.	D.	M.	Line 4388.	Line 4472.	Line H_γ.	Width.
Type B3.—*continued.*										
103 Tau.	5 2·0	+24 8	5·3	4·2	1400	− 5·4	8	10	16	3
ϱ Aur.	14·7	+41 43	5·0	1·5	150	− 0·9	4	7	10	4
23 Ori.	17·6	+ 3 27	4·8	2·3	350	− 2·9	0	2	3	10
8 Lep.	18·9	−14 1	5·0	2·8	550	− 3·7	9	8	12	3
115 Tau.	21·3	+17 53	5·1	1·7	200	− 1·4	2	4	12	7
114 Tau.	21·6	+21 51	4·7	2·2	300	− 2·7	10	8	13	4
A Ori.	25·4	+ 5 52	4·2	1·2	100	− 0·8	1	2	10	7
n^1 Ori.	26·0	+ 3 13	5·4	3·0	650	− 3·7	10	8	11	2
υ Ori.	27·1	− 7 23	4·5	2·9	600	− 4·4	6	15	15	1
120 Tau.	27·6	+18 29	5·3	3·9	1200	− 5·3	1	2	em	10
35 Ori.	28·2	+14 15	5·4	2·8	550	− 3·3	4	6	20	6
121 Tau.	29·3	+23 58	5·1	1·7	200	− 1·4	5	7	14	4
c^1 Ori.	30·4	− 4 54	4·5	1·2	100	− 0·5	3	2	15	10
125 Tau.	33·5	+25 50	4·8	2·0	250	− 2·4	2	8	10	3
ω Ori.	33·9	+ 4 4	4·4	5·0	2200	− 6·9	4	7	em	9
126 Tau.	35·5	+16 29	4·7	2·5	450	− 3·6	5	10	14	5
−1° 1004	35·8	− 1 11	4·8	1·0	100	− 0·2	7	8	10	5
55 Ori.	46·5	− 7 33	5·2	2·2	300	− 2·2	8	10	15	6
−4° 1362	6 1·6	− 4 11	5·2	2·5	450	− 3·1	3	9	20	1
f^1 Ori.	6·2	+16 9	4·8	2·8	550	− 3·9	1	1	3	10
ξ Ori.	6·3	+14 14	4·2	4·2	1400	− 6·5	2	4	10	9
−6° 1446	7·0	− 6 31	4·9	3·0	650	− 4·2	9	10	11	2
−19° 1407	13·9	−19 56	5·1	3·5	900	− 4·7	10	10	10	2
7 Mon.	14·9	− 7 47	5·0	3·0	650	− 4·1	5	6	11	9
ζ CMa.	16·5	−30 1	3·5	1·1	100	− 1·5	8	6	13	3
10 Mon.	23·0	− 4 42	4·8	0·5	50	+ 1·3	10	10	12	4
−1° 1274	28·6	− 1 9	4·8	2·2	300	− 2·6	2	3	12	10
42 Cam.	40·5	+67 41	4·9	3·8	1100	− 5·3	3	5	9	5
19 Mon.	57·9	− 4 6	4·7	0·5	50	+ 1·2	2	3	5	10
16 Arg.	8 4·5	−18 57	4·2	1·0	100	− 0·8	1	4	7	2
η Hya.	38·0	+ 3 46	4·2	0·7	50	+ 0·7	2	4	8	8
κ Hya.	9 35·5	−13 53	4·8	0·5	50	+ 1·3	3	6	10	9
90 Leo.	11 29·5	+17 21	5·6	2·0	250	− 1·4	4	8	15	8
31 Crv.	55·8	−19 6	5·1	1·8	200	− 1·4	6	7	12	5
η UMa.	13 43·6	+49 49	1·7	0·4	40	− 1·3	4	5	12	10
τ Lib.	15 32·6	−29 27	3·6	0·6	50	+ 0·1	3	5	8	8
48 Lib.	52·6	−13 59	4·5	2·8	550	− 4·2	0	1	10	15
χ Oph.	16 21·2	−18 14	4·7	2·2	300	− 2·7	7	6	em	5

TABLE V.—*continued.*

Star.	α.	δ.	m.	i.	D.	M.	Line 4388.	Line 4472.	Line H_γ.	Width.
				Type B3.—*continued.*						
u Her.	17 13·6	+33 12	var.	2·3	350	− 2·7	3	4	6	7
θ Oph.	15·9	−24 54	3·2	1·7	200	− 3·3	9	9	10	1
ι Her.	36·6	+46 4	3·6	4·8	2000	− 7·9	10	10	20	2
66 Oph.	55·3	+ 4 23	4·6	1·7	200	− 1·9	2	4	8	10
96 Her.	58·1	+20 50	4·9	3·8	1100	− 5·3	8	12	15	2
102 Her.	18 4·4	+20 48	4·2	3·5	900	− 5·6	9	10	13	3
σ Sgr.	49·1	−26 25	2·0	1·4	150	− 3·9	5	7	10	9
δ^1 Lyr.	50·2	+36 51	5·3	3·5	900	− 4·5	4	7	12	4
+50° 2708	57·7	+50 24	5·1	3·0	650	− 4·0	6	8	8	3
η Lyr.	19 10·4	+38 58	4·3	5·0	2200	− 7·4	8	9	11	1
2 Cyg.	20·2	+29 26	4·7	2·0	250	− 2·3	6	6	10	7
8 Cyg.	28·1	+34 14	4·7	4·0	1300	− 5·9	8	10	18	1
σ Aql.	34·3	+ 5 10	5·0	2·2	300	− 2·4	5	6	8	6
12 Vul.	46·8	+22 21	4·7	1·0	100	− 0·3	2	4	16	4
22 Cyg.	52·3	+38 13	4·7	2·4	400	− 3·3	6	7	13	2
25 Cyg.	56·2	+36 46	5·0	2·4	400	− 3·0	4	5	11	9
17 Vul.	20 2·5	+23 19	4·9	1·5	150	− 1·0	5	8	14	6
+25° 4165	11·0	+25 17	4·6	1·0	100	− 0·4	4	7	15	8
ω^1 Cyg.	27·0	+48 37	4·7	5·8	3100	− 7·8	3	7	10	8
51 Cyg.	39·1	+49 59	5·2	3·7	1000	− 4·8	9	10	17	1
57 Cyg.	49·7	+44 0	4·5	1·8	200	− 2·0	2	4	20	5
60 Cyg.	57·6	+45 46	5·1	1·3	100	+ 0·1	3	3	4	10
6 Cep.	21 17·3	+64 27	5·0	2·7	500	− 3·5	4	6	em	8
π^1 Cyg.	38·6	+50 44	4·6	4·2	1400	− 6·1	9	8	15	3
π^2 Cyg.	43·1	+48 51	4·1	5·0	2200	− 7·6	11	10	15	3
16 Peg.	48·5	+25 27	4·9	1·8	200	− 1·6	5	12	20	7
+55° 2639	48·8	+55 20	5·4	1·4	150	− 0·5	0	3	20	9
31 Peg.	22 16·6	+11 42	4·8	1·3	100	− 0·2	7	7	em	6
6 Lac.	26·1	+42 36	4·4	3·6	1000	− 5·6	6	7	10	2
16 Lac.	51·9	+41 4	5·4	3·2	700	− 3·8	10	13	15	4
+47° 3985	52·7	+48 9	5·0	2·8	550	− 3·7	1	3	em	10
1 Hev. Cas.	23 25·4	+58 0	4·7	1·2	100	− 0·3	6	8	10	6

IV. *Comparison with Mount Wilson Absolute Magnitudes.*

The Mount Wilson criteria (22) for the determination of absolute magnitudes of B-type stars allow for a small amount of dispersion in M within each spectral subdivision, by the classification according to the "s" and "n" characteristics. Narrow and sharp ("s") lines are associated with the more luminous stars, while broad and nebulous ("n") lines are associated with the less luminous stars. I have shown

elsewhere (23) that only a part of the observed dispersion in line-width is due to differences in M. It is known that axial rotation must and does have an appreciable effect upon line-width (24). Probably there are other factors acting in the same way. Consequently, if we group all stars of a given spectral subdivision into two groups according to line-width, the division will not be purely one of absolute magnitude. The line-width is a function of many variables, absolute magnitude being one of them: $W = f(M, x, y, z \ldots)$. Consequently, if we observe two line-widths, $W_1 > W_2$, we know only that $f_1 > f_2$, which tells us nothing concerning M_1 and M_2. Only if $x, y, z \ldots$ can all be neglected a grouping according to W is equivalent to a grouping according to M. But since they cannot be neglected it is clear that the method of spectroscopic parallaxes used at Mount Wilson and also at the Norman Lockyer Observatory gives less dispersion in M than actually exists. There is no such limitation in the calcium method. On the contrary, errors of observation as well as errors in the reduction curve tend to give a larger dispersion than is real. In comparing our absolute magnitudes with those of Adams and Joy, we expect to find that our range in M is much the greater. However, the two sets of values should show a positive correlation provided our absolute magnitudes are not merely errors of observation.

The following summary shows that our expectations are satisfied. The O-stars were not compared since the Mount Wilson absolute magnitudes for these stars differ radically from those of J. S. Plaskett and of R. E. Wilson. No significance can be attached to the B1 group since there were only five stars of that subdivision available, and since the total range in M for the Mount Wilson system amounted to only $0^m.6$. All other groups show a surprisingly good correlation between the two sets of absolute magnitudes.

TABLE VI.

Mt. W. M.	No.	Ca^+—M.	Mt. W. M.	No.	Ca^+—M.
	Type Bo.			Type B2.	
−1·2	3	−3·9	−0·9	2	−1·0
−2·4	3	−5·4	−1·9	2	−5·8
−2·9	4	−5·1	−2·3	2	−6·0
	Type B1.			Type B3.	
−2·2	4	−2·5	−0·4	4	−0·3
−2·8	1	−2·3	−0·7	13	−1·9
			−1·2	8	−3·4
			−1·5	4	−4·3
			−2·3	3	−4·1

V. *Comparison with Norman Lockyer Observatory Absolute Magnitudes.*

A similar comparison was made, using the average of the two values of M determined for each star by Edwards according to his two methods

(25). The results show a good correlation, except in subdivision B1, where the number of stars was perhaps insufficient.

TABLE VII.

Edwards's M.	No.	Ca^+−M.	Edwards's M.	No.	Ca^+−M.
	Type Bo.			Type B2.	
−2·1	5	−2·3	−1·9	8	−2·1
−2·9	5	−4·2	−2·7	7	−5·4
−3·6	5	−6·2		Type B3.	
	Type B1.		−0·7	11	−1·5
−2·3	7	3·6	−1·3	30	−2·8
−3·2	5	−3·5	−1·7	20	−2·7
			−2·3	14	−5·1

The fact that two methods so different from one another as the calcium method here employed and the classification method of the Mount Wilson and Norman Lockyer Observatories give concordant results, lends some weight to the idea that each of them contains something more than mere correlation between M and spectral type. While we have shown that the classification method suppresses part of the dispersion in M, it would be premature to claim that the large dispersion from the calcium absolute magnitudes is real.

VI. *Absolute Magnitude and Line-Width.*

The observed correlation between my absolute magnitudes and those of Adams and Joy and of Edwards can be further tested by comparing my values of M with the estimated widths of the stellar line *He* 4472. The difference ΔM between stars having narrow lines and stars having broad lines is summarized as follows :—

Type O to B2, 60 stars, $\Delta M = 1^{m}\cdot2$
Type B3, 92 stars, $\Delta M = 1^{m}\cdot9$
} Stars with narrow lines are more luminous than stars with broad lines.

As might have been expected, the differences ΔM are larger than those of Adams and Joy, though they are of the same order of magnitude as the values obtained by Edwards. This is probably due to the fact that Edwards has used a finer classification of line-widths. His values are :

Type Bo to B2 : $\Delta M = 0^{m}\cdot9$.
Type B3 : $\Delta M = 1\cdot5$.

We conclude that line-width is rather strongly correlated with absolute magnitude, but obviously there is no one-to-one relationship, since line-width depends also upon rotation, and probably also upon other factors not related to absolute magnitude.

VII. *Absolute Magnitude and Intensities of Helium Lines.*

An attempt was made to find whether any relationship between the intensities of the helium lines and the absolute magnitudes could be established. From the observations it appears that, aside from line-width, no very marked correlation exists. There is, however, in the earlier types, O to B2, an interesting though small correlation between absolute magnitude and the difference in intensity of the two helium lines 4472 and 4388. This is illustrated in fig. 2. The differences are largest for the most luminous stars. There is a tendency for the correlation to decrease as we proceed from type O to B2. In B3 no correlation is found, in spite of the large number of stars. It is difficult to interpret this correlation physically, but it is probably caused by the properties of the helium atoms. In any case, the fact that an appreciable correlation exists in types O to B2, lends additional weight to our absolute magnitudes.

An inspection of the observational data shows that a large difference in the intensities of the two helium lines is usually associated with very strong Ca^+ lines (9 α Camelop. ; 9 Cephei ; λ Orionis ; 19 Cephei), while weak Ca^+ lines are peculiar to stars where the two lines are nearly equal in strength (β Cephei ; α Virginis). But occasionally the opposite is observed. Thus in χ^2 Orionis the two helium lines are nearly equal, while the unusually strong Ca^+ lines suggest great distance and consequently also great luminosity ; another example is 55 Cygni.

VIII. *The Theoretical Line-contour.*

The success attained by J. Q. Stewart and by A. Unsöld in the application of the classical electron theory to absorption in a stellar atmosphere suggests that a similar application could be made with respect to the detached lines of Ca^+. The classical theory, which considers the atom as a simple isotropic harmonic oscillator with radiation damping, leads to the following known expression for the absorption coefficient per cm. :

$$\sigma = \frac{2\pi e^4 \lambda_0^2 N f}{3m^2c^4(\lambda - \lambda_0)^2} = C\frac{N}{(\lambda - \lambda_0)^2} \quad . \quad . \quad (9)$$

In this expression :

e = Charge of electron.
m = Mass of electron.
λ_0 = Central wave-length of line.
c = Velocity of light.
f = Oscillator strength.
N = Number of atoms per cm.^{3}.
λ = Wave-length.

The mechanism of absorption in interstellar matter is not the same as in a stellar atmosphere. Professor Eddington has (26) pointed out that since the light-source in this case is a point, the absorbed energy

is taken out of the ray and re-emitted in all directions, contributing to the general illumination of the sky, but not re-enforcing the original ray. Accordingly the intensity of the dimmed light is not that given

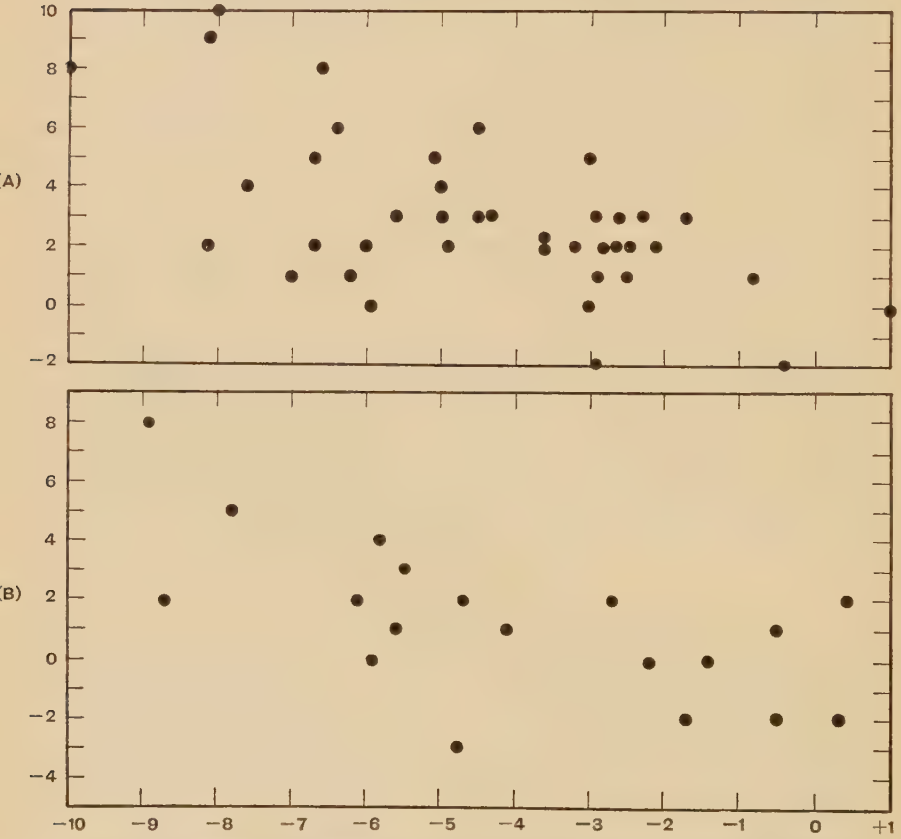

FIG. 2.—Difference in intensity of *He* λ4472 and *He* λ4388 (ordinates) and absolute magnitude from Ca^+ lines (abscissæ). (A) Spectral types O to B1. (B) Spectral type B2.

by the Schuster-Schwarzschild approximation (which was used by Unsöld), but is that corresponding to an exponential decrease of light:

$$I/I_0 = e^{-\sigma D} \quad . \quad . \quad . \quad . \quad (10)$$

The intensity I_0 refers to the continuous spectrum in the immediate vicinity of the line. Consequently the theoretical contour of the interstellar Ca^+ line is given by

$$I = I_0 e^{-C\frac{N.D}{(\lambda-\lambda_0)^2}} \quad . \quad . \quad . \quad . \quad (11)$$

It would be interesting to compare this equation with the actual contour of the line. However, the resolving power of our present-day stellar spectrographs, giving a dispersion of about 20 A. per mm. at K, is not sufficient to yield the true contour of the line. At my request Dr. A. Pogo of the Yerkes Observatory has measured the width of the detached K line in 122 stars. The mean width is 0·46 A., which agrees closely with the limiting value imposed by the optical properties of the spectrograph. Furthermore, there is practically no increase in width with increase of intensity.

TABLE VIII.

Centr. Int.	No.	Mean Width.	Centr. Int.	No.	Mean Width.
0·0–1·0	7	0·45 A.	3·0–4·0	20	0·50 A.
1·0–1·5	18	0·48	4·0–5·0	12	0·49
1·5–2·0	16	0·39	5·0–6·0	8	0·47
2·0–2·5	22	0·44	6·0–9·0	6	0·54
2·5–3·0	13	0·42			

It is clear that the observed line-contour is not that given by formula (11), but is imposed by the finite resolving power of the spectrograph. The total absorbed energy is redistributed in the observed line in such a way that the width is kept approximately constant and equal to the minimum imposed by the spectrograph. Obviously this takes place at the expense of the central intensity. Consequently we observe a decrease of central intensity for the nearer stars, in spite of the fact that the true contour (11) requires zero central intensity for all stars.

The observed contour of a narrow spectral line, affected by the optical properties of the spectrograph, is closely represented by the error curve :

$$I = 1 - (1 - I_c)e^{-\kappa^2(\lambda-\lambda_0)^2} \quad . \quad . \quad . \quad (12)$$

I_c represents the central intensity of the line. The total absorbed energy, as observed, is accordingly

$$E_{obs} = \int_{-\infty}^{+\infty}(1 - I)d\lambda = \int_{-\infty}^{+\infty}(1 - I_c)e^{-\kappa^2(\lambda-\lambda_0)^2}d\lambda$$

$$= \frac{(1 - I_c)}{\kappa}\int_{-\infty}^{+\infty}e^{-x^2}dx = \frac{(1 - I_c)\sqrt{\pi}}{\kappa}.$$

In our case the value of κ is given by the observed width, W = 0·46 A. From (12) we have

$$\frac{1 - I}{1 - I_c} = e^{-\kappa^2(\lambda-\lambda_0)^2}.$$

The width, W, is measured at a point on the contour where $(1 - I)$ corresponds to the minimum observable contrast on the photographic plate. From previous determinations I have taken this to be equal to 0·04. Consequently

$$e^{-\frac{\kappa^2 W^2}{4}} = \frac{0{\cdot}04}{1 - I_c}.$$

Solving for κ we get $$\kappa = \frac{2}{W}\sqrt{\log_e\left\{\frac{1-I_c}{0{\cdot}04}\right\}}.$$

Accordingly the total absorbed energy is

$$E_{obs} = \frac{(1-I_c)\sqrt{\pi}\,.\,W}{2\sqrt{\log_e\left\{\frac{1-I_c}{0{\cdot}04}\right\}}} = \frac{0{\cdot}23\sqrt{\pi}(1-I_c)}{\sqrt{\log_e\left\{\frac{1-I_c}{0{\cdot}04}\right\}}} \quad . \quad (13)$$

It should be remembered that in this expression I_c is measured in units of the intensity of the continuous spectrum, while the observed quantities are expressed in stellar magnitudes.

The theoretical value of E is obtained from (11). Assuming, in accordance with the preceding, that the intensity of the continuous spectrum $I_0 = 1$, we have

$$E_{theor} = \int_{-\infty}^{+\infty}\left\{1 - e^{-C\frac{ND}{(\lambda-\lambda_0)^2}}\right\}d\lambda \quad . \quad . \quad (14)$$

Substituting

$$\frac{\lambda-\lambda_0}{\sqrt{CND}} = x$$

we obtain

$$E_{th} = \sqrt{CND}\int_{-\infty}^{+\infty}\left(1 - e^{-\frac{1}{x^2}}\right)dx \quad . \quad . \quad (15)$$

The integral in (15) can best be transformed by changing from the X to the Y co-ordinate. Obviously

$$\int_0^{\infty}\left(1 - e^{-\frac{1}{x^2}}\right)dx = \int_0^1 \frac{dy}{\sqrt{\log_e 1/y}} = \sqrt{\pi}.$$

The latter integral dates back to Euler and to Legendre. Consequently

$$E_{th} = 2\sqrt{\pi}\sqrt{CND} \quad . \quad . \quad . \quad . \quad (16)$$

The theoretical E_{th} and the observed E_{obs} must be identical, so that

$$2\sqrt{C}\sqrt{ND} = \frac{0{\cdot}23\,(1-I_c)}{\sqrt{\log_e\left(\frac{1-I_c}{0{\cdot}04}\right)}} \quad . \quad . \quad . \quad (17)$$

This is our theoretical relationship between D and I_c. The value of C can be computed from (10): $C = 1{\cdot}7 \times 10^{-18}$. N is unknown, and must be determined with the help of the empirical curve in fig. 1. We assume that for a Ca^+ intensity of 5 arbitrary units, corresponding to $(1-I_c) = 0{\cdot}37$, the distance D = 2200 parsecs $= 6{\cdot}8 \times 10^{21}$ cm. Substituting these numerical values in (17) we find

$$N = 7{\cdot}5 \times 10^{-8} \text{ atoms per cm.}^3.$$

As the weight of a calcium atom is $6{\cdot}6 \times 10^{-23}$ gr., we obtain for the density of Ca^+ in space

$$\rho_{Ca^+} = 5 \times 10^{-30} \text{ gr./cm.}^3,$$

a value agreeing well with Eddington's preliminary estimate.

With our value of N we may now compute the theoretical distance-intensity curve from equation (17). The results are given in Table IX.

TABLE IX.

Int. in Arbitrary Units.	$(1-I_c)$.	D (Parsecs).	Int. in Arbitrary Units.	$(1-I_c)$.	D (Parsecs).
1	0·09	350	5	0·37	2100
2	0·17	700	6	0·42	2600
3	0·24	1100	7	0·47	3100
4	0·31	1600	8	0·52	3600

The agreement with the empirical curve is less satisfactory than might have been expected. I am inclined to attribute the disagreement to the uncertainty of the calibration of intensities. Indeed, this calibration is of no importance whatever as long as we consider only the empirical curve. But in the theoretical curve a fairly exact calibration is essential. Now the stronger intensities can be calibrated by means of the spectro-photometer, and my values for these lines have been checked several times. But for the fainter lines the calibration is extremely uncertain. If my scale had been such that the units corresponded to equal steps in $(1 - I_c)$ instead of in stellar magnitudes, the agreement would have been better.

It is of interest to note that for the strongest lines (intensity 10 on my arbitrary scale) the total number of Ca^+ atoms, $ND = 10^{15}$. If we assume with Eddington that the proportion of singly ionized calcium atoms to the whole number of calcium atoms is approximately 10^{-3}, we obtain for the total number of all calcium atoms, 10^{18}. Finally, if the proportion of calcium in space (by atoms) is 10^{-2}, the total number of all atoms of all elements turns out to be about 10^{20}. If we contrast this with Loschmidt's number, $2{\cdot}67 \times 10^{19}$, we arrive at a good idea of the state of rarefaction of interstellar matter. The quantity of matter contained in 1 cm.^{3} of air at normal temperature and pressure is nearly equal to the quantity of matter in a column of 1 cm.^{2} cross-section extending through something like 5000 parsecs.

It will also be noted that for a strong detached line the natural width at the point of minimum contrast is very nearly equal to that imposed by the spectrograph. If, for example, $ND = 10^{15}$, then from (11)

$$I = 0{\cdot}95 = e^{-1{\cdot}7\times10^{-18}\frac{10^{15}}{(\lambda-\lambda_0)^2}}.$$

Solving this for $(\lambda - \lambda_0)$ we obtain for the total width

$$W = 2(\lambda - \lambda_0) = 0{\cdot}3 \text{ A.}$$

The limit imposed by the spectrograph is about 0·4 A. It would therefore seem that for the strongest lines a dispersion slightly greater than that used at the Yerkes Observatory should enable us to obtain the real line-contours. It is even probable that the strongest observed lines are just at the limit where the real contour begins to be observable with our dispersion. It is noticeable from a visual inspection of the plates that the contours of the detached lines are not identical with those of ordinary stellar K lines, even if the central intensities are the same. The detached lines are sharper and steeper on the sides, while the stellar lines look slightly fuzzy at the edges. This, if real, would agree with the fact that our contour given by (11) markedly differs from that of the Schuster-Schwarzschild approximation used for stellar lines by Unsöld (27) and Miss Payne (28):

$$I/I_0 = \frac{1}{1 + \sigma D} \quad . \quad . \quad . \quad . \quad (18)$$

where σ has its usual meaning given by (9). The difference in the shape of the two curves is illustrated in fig. 3, showing graphs of the two functions:

$$I = e^{-\frac{1}{x^2}} \quad \text{and} \quad I = \frac{1}{1 + \frac{1}{x^2}} = \frac{x^2}{x^2 + 1}.$$

A word may be added concerning the evaluation of ND for lines produced according to the mechanism of (18). Unsöld and Miss Payne used actual line-contours. For lines which are too narrow to give reliable contours, the total absorbed energies may be used as we have done for the detached lines. For a Schuster-Schwarzschild line the total absorbed energy is

$$E' = \int_{-\infty}^{+\infty} \left\{ 1 - \frac{1}{1 + \frac{CND}{(\lambda - \lambda_0)^2}} \right\} d\lambda = \sqrt{CND} \int_{-\infty}^{+\infty} \frac{dx}{x^2 + 1} = \pi . \sqrt{CND}.$$

The ratio of the absorbed energies in the two mechanisms (11) and (18) is

$$\frac{E}{E'} = \frac{2}{\sqrt{\pi}} = 1{\cdot}13.$$

The relationship $E \sim \sqrt{ND}$ (which is characteristic for both mechanisms) was shown to hold for the lines of the solar spectrum by M. Minnaert and B. van Assenbergh (29). This reminds us of the results of Gouy and of others on the total intensity of a flame. Their experimental results were discussed and explained by H. A. Wilson (30). Wilson pointed out also that the square-root law should hold for processes of absorption—in good agreement with our formula (16). However, his theoretical expression (31) differs from ours. He finds

$$\log E \sim \sqrt{ND}.$$

This law is confirmed by laboratory measurements, by A. Ll. Hughes and A. R. Thomas (32), of the absorption of the mercury line $\lambda 2536$ A.

However, the case of an interstellar gas producing an absorption line in a stellar spectrum differs in one important respect from the experimental arrangement of Hughes and Thomas. In the latter case the emitting light-source was a mercury arc, while mercury vapour was used in the absorbing cell. Accordingly the absorption line was superimposed over an emission line of the same element. Wilson adopts for the distribution of light in the emission line the expression $Ae^{-\alpha^2\Delta^2}d\Delta$,

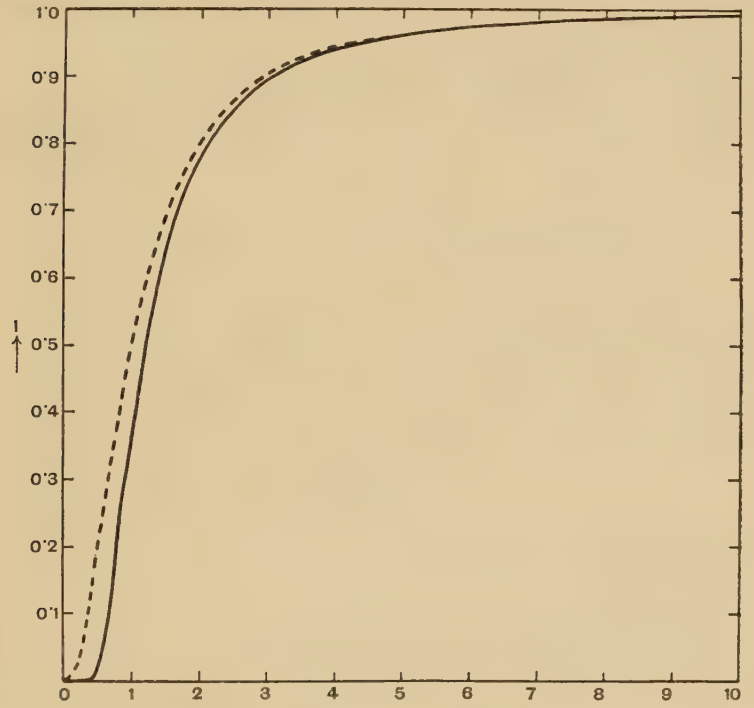

FIG. 3.—Full line: $I=e^{-\frac{1}{x^2}}$. Broken line: $I=\frac{x^2}{x^2+1}$.

where A and α are constants and Δ is the frequency-interval. Consequently he finds for the ratio of the transmitted to incident light:

$$\frac{E}{E_0} = \frac{\int_{-\infty}^{+\infty} Ae^{-\alpha^2\Delta^2-\sigma D}d\Delta}{\int_{-\infty}^{+\infty} Ae^{-\alpha^2\Delta^2}d\Delta}.$$

This reduces to

$$E/E_0 = e^{-\frac{\alpha \cdot k}{m}\sqrt{ND}},$$

or

$$\log E \sim \sqrt{ND}.$$

In the case of a detached line the incident light is that of the star's continuous spectrum. Consequently I_0 = const. = 1. This brings us back to our earlier equation (14), leading to the law $E \sim \sqrt{ND}$. The difference arising from the character of the background spectrum was also discussed by R. Ladenburg and F. Reiche (33). According to them the case of an absorption line superposed over a continuous emission spectrum gives the same relationship to ND as the total brightness of an emission line or flame ($E \sim \sqrt{ND}$ if ND is large (37), and $E \sim ND$ if ND is small).

It is strange that, as Wilson has shown, the empirical curve in fig. 1 closely agrees with the law $\log E \sim \sqrt{ND}$. From the preceding discussion it would seem that this is only a coincidence due to errors in the calibration of the intensities, and that the real relationship is given by $E \sim \sqrt{ND}$.

The density which we have derived for Ca^+ in space is about 10^2 times greater than that determined by Gerasimovič and Struve (34) by means of Milne's absorption coefficient for Ca^+. In view of the great uncertainty of many of the data used—the stellar distances, for example—this is not unreasonable. The order of magnitude of the densities involved is about the same, averaging about 10^{-25} gr./cm.^{3} for all atoms of all elements.

I am greatly indebted to Professor A. S. Eddington for numerous suggestions in connection with this work.

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(37) From the data of H. A. Wilson it would appear that this law holds for ND greater than about 10^{13}. As we are dealing here with values of the order of 10^{14} or 10^{15} the relationship of formula (16) is thus confirmed.

Cambridge:
1929, *April* 1.

On the Dwarf Nature of the Spectroscopic Binaries.

By Dr. E. A. Kreiken.

1. As a rule the spectroscopic binaries are very bright, and therefore it is generally assumed, though never proven, that they are giants. There is no doubt a selection of the apparently very bright stars. Nevertheless, evidence will be given in the present paper that the spectroscopic binaries are dwarfs.

At present two very good catalogues of spectroscopic binaries are available, viz., "The Third Catalogue of Spectroscopic Binaries," by J. Moore (*Lick Observatory Bulletin*, No. 355, p. 141, 1924), and the one by A. Beer (*Veröff. Berlin-Babelsberg*, Band V., Heft 6, p. 80, 1927). From these two catalogues I borrowed all systems with known spectra of both the primary and secondary star. The numerical values $m_1 \sin^3 i$ and $m_2 \sin^3 i$ of these stars are known. If the star is at the same time an Algol or eclipsing variable, the masses m_1 and m_2 are known directly. Both groups of stars are entered in Table I. For more details concerning these stars I refer to the two catalogues mentioned before. The mean value $\overline{\sin i}$ should be larger than the one obtained from a haphazard distribution of the inclinations i, this being due to the fact that for small values of i, the variability of the radial velocity cannot easily be detected. Usually the mean value of $\sin i$ is put equal to $2/3 = 0{\cdot}667$, and the masses of the stars are derived by multiplying the numerical values of $m_1 . \sin^3 i$ and $m_2 . \sin^3 i$ by $\left(\frac{1}{0{\cdot}667}\right)^3$. In this way masses of these stars are obtained which are comparable with those of the giants. In the following I tried to fix the exact value of $\overline{\sin i}$. It appeared that the value of $\overline{\sin i}$ is much larger than the one used up till now. If this new value of $\overline{\sin i}$ is applied to the functions $m_1 . \sin^3 i$ and $m_2 . \sin^3 i$ the masses of the spectroscopic binaries do not agree any longer with those of the giants, but correspond to those of the main sequence of dwarfs.

TABLE I.

List of Objects used in the Present Investigation.

No.	Boss No.	α (1900).	δ (1900).	App't. Mag.	$m_1 \sin^3 i$.	$m_2 \sin^3 i$.	Spectral Types.	Remarks.
		h m	° ′					
1	46	0 12·4	+50 53	6·12	35⊙	32⊙	O8 : O8	
2	TV Cas.	13·9	+58 35	Var.			B9 : –	Algol var. m_1=1·83⊙ m_2=1·01⊙. Max. ligh 7·27.
3	116	30·1	− 4 9	5·24	0·59⊙	0·73⊙	F8 : G1	
4	145	37·9	+46 29	5·02	1·35⊙	1·34⊙	A5 : A5	
5	175	44·3	+40 32	4·42	1·50⊙	1·10⊙	B3 : B3	
6	373	1 35·7	+25 14	6·26	1·16⊙	1·06⊙	F3 : F3	
7	384	37·4	+50 11	4·19	1·1⊙	2·5⊙	Bo : –	No single orbit or com bination of orbit completely represent the observed velocit curve.
8	497	2 6·6	+29 50	5·20	1·12⊙	1·12⊙	Go : Go	
9	497	2 6·6	+29 50	6·40	0·906⊙	0·855⊙	F2 : –	Fainter comp. of visua double.
10	RZ Cas.	39·9	+69 13	Var.			Ao : –	Eclips. var. m_1=0·58⊙ m_2=1·58⊙. Max. ligh 6·43.
11	708	3 1·7	+40 34	Var.			B8 : –	Two solutions. Fron the first m_1=0·370 m_2=0·18⊙. From th second m_1=4·72⊙ m_2=0·47.
12	H.D. 19820	6·2	+52 9	7·4	18·9⊙	9·2⊙	O8 : O8	
13	816	29·4	−21 58	4·3	2·76⊙	2·87⊙	B8 : B8	Max. light 2·4.
14	920	55·1	+12 12	Var.			B3 : –	Eclips. var. m_1=2·5⊙ m_2=1·00⊙; m_3=0·4⊙
15	H.D. 25833	4 0·6			4·86⊙	4·29⊙	B3 : B3	Max. light 3·76.
16	986	10·7	+50 3	4·57	0·86⊙	0·24⊙	A2 : A2	
17	1001	14·1	−34 3	3·59	0·558⊙	0·548⊙	B9 : –	
18	1076	30·2	+ 9 57	4·38	2·7⊙	1·3⊙	A3 : F	
19	1213	5 1·8	−44 47	5·19	2·8⊙	2·2⊙	B9 : B9	
20	1246	9·3	+45 54	0·21	1·19⊙	0·94⊙	Go : Go	Visual orbit from inter ferometer measures.
21	1275	14·8	+29 28	5·72	1·71⊙	1·50⊙	A1 : A1	
22	1301	19·4	− 2 29	3·44	11·2⊙	10·6⊙	B1 : B1	
23	1314	21·6	+ 3 0	4·66	5·53⊙	4·19⊙	B2 : B2	
24	1457	47·0	+27 35	4·54	0·63⊙	0·44⊙	Ao : A5	
25	1478	52·2	+44 56	2·07	2·21⊙	2·17⊙	Ao : Ao	

TABLE I.—*continued.*

No.	Boss No.	α (1900). h m	δ (1900). ° ′	App't. Mag.	$m_1 \sin^3 i$.	$m_2 \sin^3 i$.	Spectral Types.	Remarks.
26	1492	5 54·3		5·10	0·91⊙	0·78⊙	A5 : A5	
27	1501	56·9	+ 9 39	4·19	3·8⊙	1·2⊙	A2 : –	
28	1515	59·7	+38 29	5·13	1·354⊙	1·113⊙	A3 : –	
29	1646	6 25·9	+32 32	5·98	2·2⊙	1·9⊙	Ao : –	
30	B.D. 1309	32·0	+ 6 13	6·06	75·6⊙	63·6⊙	Oe5 : Oe5	
31	1732	40·7	−16 35	−2·0	0·81⊙	0·31⊙	Ao : Fo	
32	1945	7 22·7	−22 53	5·7	1·05⊙	0·85⊙	F5 : F5	
33	1979 (3)	28·2	+32 6	1·99	0·63⊙	0·57⊙	M1 : M1	
34	1979	28·2	+32 6	2·85			Ao : –	Castor. $m_1+m_2=6·5$. In the following it was supposed $m_1=m_2$.
35	H.D. 61859	36·2	+34 14	6·00	1·53⊙	1·32⊙	F3 : F3	
36	2075	47·1	−13 38	5·5	0·67⊙	0·29⊙	F8 : –	
37	2354	8 41·5	+ 6 47	3·48			F8 : –	Visual double. $m_1=1·75⊙$; $m_2=1·58⊙$.
38	2484	9 10·8	+47 14	5·70	1·48⊙	1·27⊙	A2 : –	
39	S Ant.	27·9	−28 11	Var.			Fo : Fo	Eclips. var. $m_1=0·75⊙$; $m_2=0·42⊙$. Max. light 6·3.
40	2602	35·8	+10 21	3·76	1·302⊙	1·116⊙	F5 : Ao	
41	W UMa	36·7	+56 25	Var.			F8 : F8	Eclips. var. $m_1=0·69⊙$; $m_2=0·49⊙$
42	2830	10 33·1	−47 42	4·06	0·281⊙	0·235⊙	F2 : –	
43	2900	48·2	+43 43	4·84	3·50⊙	0·59⊙	Ao : Ao	
44	2987	11 13·7	+38 44	4·78	0·123⊙	0·085⊙	A2 : F	
45	3210	12 14·8	− 0 7	4·00	1·10⊙	0·77⊙	Ao : Ao	
46	H.D. 107760	17·9	+37 48	8·2	0·803⊙	0·704⊙	G5 : –	
47	3363	40·7	+ 8 13	5·24	2·3⊙	0·3⊙	A5 : –	
48	3354	48·3	+83 58	6·1	2·47⊙	2·08⊙	Ao : Ao	
49	RS CVn	13 6·0	+36 28	Var.	1·3⊙	1·3⊙	F8 : Ko	Eclips. var. Max. light 7·85.
50	3476	19·9	−10 38	1·21	9·6⊙	5·8⊙	B2 : B2	
51	3474	19·9	+55 27	2·40	1·70⊙	1·62⊙	A2 : A2	
52	3635	14 5·8	+25 34	4·82	1·36⊙	1·29⊙	F5 : F5	
53	3735	32·8		0·0	1·13⊙	0·97⊙	G4 : K4	
54	3793	46·3	+49 8	6·8	1·27⊙	1·03⊙	F5 : F5	
55	3825	55·6	− 8 7	Var.			Ao : –	Eclips. var. $m_1=0·87⊙$; $m_2=0·63⊙$. Max. light 7·9.

TABLE I.—*continued.*

No.	Boss No.	α (1900). h m	δ (1900). ° ′	App't. Mag.	$m_1 \sin^3 i$.	$m_2 \sin^3 i$.	Spectral Types.	Remarks.
56	U CrB.	15 14·1	+32 1	Var.			B3 : B3	Eclips. var. $m_1=4{\cdot}27\odot$ $m_2=1{\cdot}63\odot$ Max. ligh 7·9.
57	3988	35·6	+36 58	5·1	13·3⊙	13·1⊙	B8 : B8	
58	4086	59·6	−19 32	2·90	13·0⊙	8·3⊙	B1 : B1	
59	4138	16 10·9	+34 7	5·7	1·07⊙	0·94⊙	F8 : F8	
60	H.D. 149632	30·9	+17 15	6·27	2·19⊙	1·35⊙	A0 : A0	
61	4246	37·5	+31 47	2·8	0·62⊙	0·27⊙	G0 : –	
62	4247	37·6	+27 7	6·1	1·11⊙	0·99⊙	F2 : F2	
63	4328	56·5	+31 4	3·92	1·6⊙	1·0⊙	A0 : A0	
64	U Oph.	17 11·4	+ 1 19	Var.			B3 : B3	Eclips. var. $m_1=5{\cdot}31\odot$ $m_2=4{\cdot}66\odot$. Max. ligh 6·0.
65	*u* Her.	13·6	+33 12	Var.			B3 : B3	Eclips. var. $m_1=7{\cdot}5\odot$ $m_2=2{\cdot}9\odot$. Max. ligh 4·6.
66	TX Her.	15·4	+42 0	Var.			A2 : A2	Eclips. var. $m_1=2{\cdot}04\odot$ $m_2=1{\cdot}77\odot$. Max. ligh 7·96.
67	4423	21·3	− 5 0	4·61	0·88⊙	0·84⊙	F0 : –	
68	Z Her.	53·6	+15 9	Var.			F2 : F2	Eclips. var. $m_1=1{\cdot}6\odot$ $m_2=1{\cdot}3\odot$. Max. ligh 7·13.
69	4571	18 0·4	+ 2 31	4·3	0·68⊙	0·37⊙	K0 : K5	
70	4602	7·5	+79 59	6·18	0·457⊙	0·413⊙	F5 : F5	
71	4622	12·9	+56 33	6·41	1·043⊙	1·013⊙	F0 : F0	
72	4643	17·1	+29 48	5·54	1·72⊙	1·18⊙	A3 : A3	
73	RX Her.	26·0	+12 30	Var.			A0 : A3	Eclips. var. $m_1=0{\cdot}96\odot$; $m_2=0{\cdot}91\odot$. Max. light 7·0.
74	205 Dra.	45·0	+49 19	7·18	1·48⊙	1·47⊙	F2 : A5	
75	4776	46·4	+33 15	Var.			B8 : B5	β Lyræ $m_1=9{\cdot}65\odot$; $m_2=20{\cdot}91\odot$. Max. light 3·44.
76	4788	49·6	+75 19	5·37	0·95⊙	0·90⊙	A0 : A0	
77	H.D. 178619	19 4·2	+16 42	6·46	1·26⊙	1·26⊙	F5 : F5	
78	RS Vul.	13·4	+22 16	Var.			B8 : B9	Eclips. var. $m_1=5{\cdot}4\odot$; $m_2=1{\cdot}69\odot$. Max. light 7·4.
79	4934	16·0	−16 9	4·58	260⊙	54⊙	F2 : B8	Doubtful whether these masses are real.
80	Z Vul.	17·5	+25 23	Var.			B3 : B3	Eclips. var. $m_1=5{\cdot}24\odot$; $m_2=2{\cdot}36\odot$. Max. light 7·3.

TABLE I.—*continued.*

No.	Boss No.	α (1900).	δ (1900).	App't. Mag.	$m_1 \sin^3 i$.	$m_2 \sin^3 i$.	Spectral Types.	Remarks.
		h m	° ′					
81	4947	18 19·8	+16·45	6·03	0·91⊙	0·65⊙	Ao: –	
82	5018	34·3	+ 5 10	5·17	5·3⊙	4·4⊙	B8: B8	
83	5026	36·4	+54 44	5·86	1·85⊙	1·83⊙	F5: F5	
84	H.D. 191201	20 3·6	+35 26	7·12	13·8⊙	12·9⊙	O9: O9	
85	5171	6·1	– 1 7	3·37	0·52⊙	0·38⊙	Ao: –	
86	5173	6·4	+26 36	5·46	2·27⊙	2·06⊙	A2: –	
87	*y* Cyg.	48·1	+34 17	Var.			B2: B2	Eclips. var. m_1=16·6⊙; m_2=15·3⊙. Max. light 7·1.
88	5375	49·7	+44 0	4·68	1·79⊙	1·67⊙	B3: B3	
89	H.D. 205539	21 30·9	+27 46	6·35	0·97⊙	0·77⊙	F3: F3	
90	77 Cyg.	38·4	+40 37	5·8	0·96⊙	0·95⊙	Ao: Ao	
91	5591	39·9	+28 19	6·90	1·19⊙	1·17⊙	A9: A9	
92	5592	40·1	+25 11		9·66⊙	3·73⊙	F3: F5	
93	5764	22 16·9	+46 2	4·66	0·87⊙	0·71⊙	B5: B5	
94	H.D. 216014	44·2	+64 32	6·83	14·2⊙	12·4	Bo: Bo	
95	H.D. 218440	23 3·0	+59 13	6·28	2·80⊙	2·24⊙	B3: B3	
96	6142	50·5	+56 51	6·05	18·5⊙	12·7⊙	Bo: –	
97	6148	52·1	+55 9	5·69	1·704⊙	1·671⊙	F5: F5	

As this is directly opposed to the common view, I have tried to obtain further evidence by comparing the distances derived from Eddington's mass-luminosity relation with those obtained from trigonometrical and spectroscopic parallaxes. The theoretical distances derived from the assumption that the spectroscopic binaries are giants do not agree at all with the observed ones. On the other hand, the theoretical distances derived from the dwarf masses agree perfectly. Therefore I think that the spectroscopic binaries really are dwarf stars. As the question is rather important, many details have been given in order to settle this question definitely.

2. *The Relation between* $m \cdot \sin^3 i$ *and Spectral Type.*—In fig. 1 the logarithms of the observed values $m_1 \cdot \sin^3 i$ and $m_2 \cdot \sin^3 i$ are plotted against spectral type. Only when the spectral type of the fainter component has been stated definitely have the values $m_2 \sin^3 i$ been entered into the diagram. Two points indicating the two components of one and the same system have been connected. The Algol or eclipsing variables have not been inserted.

The dotted curve in the figure indicates the mean relation between $m \cdot \sin^3 i$ and spectral type. I cannot think there is any reason why the mean value of $m \cdot \sin^3 i$ should systematically depend on spectral type. Therefore the actual relation between mass and spectral type will be

obtained by shifting the dotted curve to the left over an interval corresponding to the logarithm of $\overline{\sin^3 i}$.

So our first problem is the determination of the value of $\log . \overline{\sin^3 i}$. It is clear that when our curve corresponds to the giant series this value

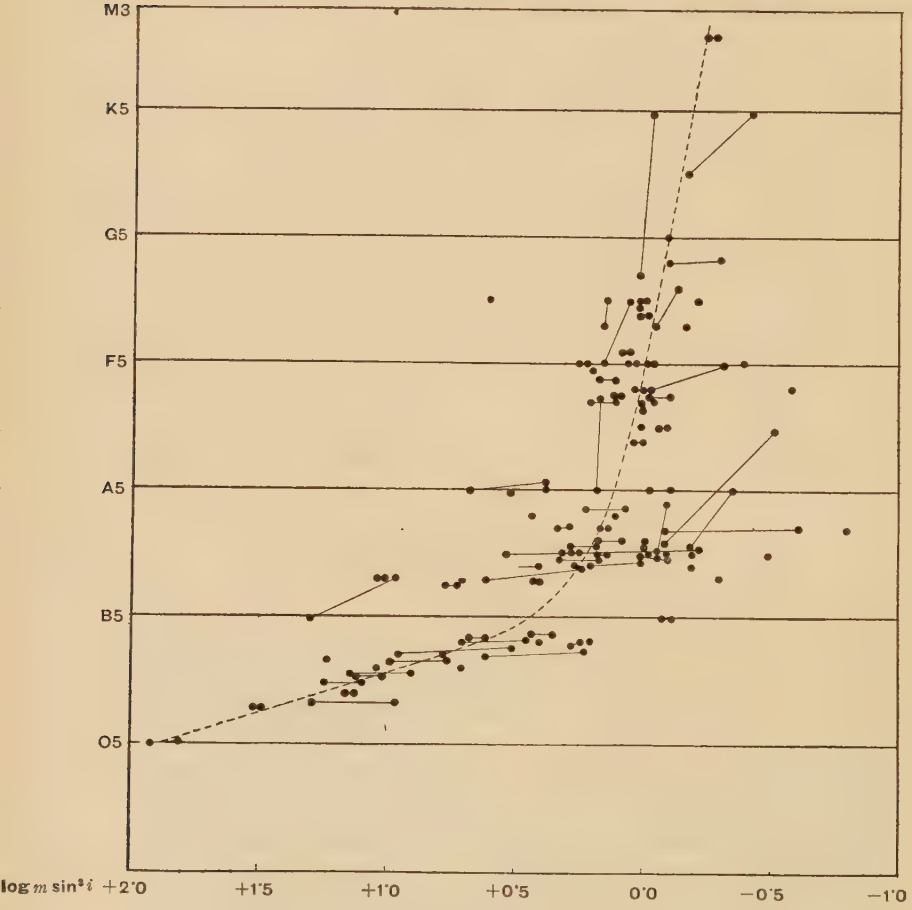

FIG. 1.—Relation between $\log m \sin^3 i$ and spectral type.

will be smaller than when it corresponds to the dwarf sequence. We will therefore compare our curve with those obtained from both series. The results have been given in Table III. (giants) and Table IV. (dwarfs). The masses of the giant stars were computed from the absolute magnitudes of the giants in the Russell diagram by E. P. Hubble.* The details of this computation are given in Table II. In the first column of this

* *Carnegie Institution Year-Book*, No. 20, p. 270, 1921.

table the spectral type has been entered and in the second the absolute magnitude. The third contains the effective temperatures of the different spectral types according to Miss Payne.* In the fourth the bolometric reduction has been given.† The bolometric magnitude appears in the fifth column. In the sixth I have given the corrections due to the temperature term, $2 \cdot \log T_e/5200$.‡

TABLE II.

Masses of Giant Stars computed from Absolute Magnitudes.

Spectral Type.	Mean Abs. Mag.	T_e.	Bol. Corr'n.	Bol. Mag.	$+2 \log \frac{T_e}{5200}$.	Corr. Mag.	Mass acc. to Eddington.	log Mass.
Bo	−1·0	20000	−1·96	−2·96	+1·17	−1·79	7·36	0·87
Ao	0·0	10000	−0·50	−0·50	+0·57	+0·07	3·63	0·56
Fo	0·0	7500	−0·04	−0·04	+0·32	+0·28	3·38	0·53
Go	−2·0	5600	−0·03	−2·03	+0·06	−1·97	7·97	0·90
Ko	+1·5	4000	−0·34	+1·16	−0·23	+0·93	2·72	0·43

TABLE III.

Comparison of the Curve in Fig. 1 with the Giant Curve (Smoothed Values of Table II.).

Spectral Type.	log Mass. (Giants).	log $m \sin^3 i$ from Fig. 1.	Difference (− log $\sin^3 i$).
Bo	0·87	1·10	−0·23
B5	0·65	0·44	+0·21
Ao	0·56	0·22	+0·34
A5	0·54	0·11	+0·43
Fo	0·52	0·05	+0·47
F5	0·45	9·99	+0·46
Go	0·44	9·95	+0·49
G5	0·43	9·90	+0·53

With the corrected magnitudes in column 7 (Table II.) we obtain from Eddington's table 14§ the masses of the giants as given in column 8. Finally, the ninth column contains the logarithms of these masses. The latter were plotted against spectral type, and a smooth curve was drawn through the points obtained in this way. These smoothed values were used in Table III.

The masses of the dwarf stars in Table IV. were taken from the investigation by A. Brill on visual binaries.‖

* "Stellar Atmospheres," *Harvard Monographs*, table ix. p. 33.
† Nutting, *Phil. Mag.*, p. 304, Feb. 1915.
‡ Eddington, *The Internal Constitution of the Stars*, p. 137.
§ *Loc. cit.*, p. 137.
‖ *Veröff. Berlin-Babelsberg*, Band VII. Heft 1, table 7, p. 25, 1927; comp. also W. Rabe, *A.N.*, 5525.

TABLE IV.

Comparison of the Curve in Fig. 1 with the Dwarf Curve according to A. Brill.

Spectral Type.	log Mass (Dwarfs).	log $m \sin^3 i$ from Fig. 1.	Difference ($-\log \overline{\sin^3 i}$).
O5	2·00	1·90	+0·10
Bo	1·00	1·10	−0·10
B5	0·65	0·44	+0·21
Ao	0·44	0·22	+0·22
A5	0·30	0·11	+0·19
Fo	0·18	0·05	+0·13
F5	0·08	9·99	+0·09
Go	0·00	9·95	+0·05
G5	−0·08	9·90	+0·02
Ko	−0·16	9·86	−0·02

In Tables III. and IV. the masses obtained from our Curve I. are compared with those of the giants and dwarfs.

Let us consider in the first place Table III. The differences between the logarithms of the masses of the giant stars and those of $m \,.\, \sin^3 i$ of our curve have been given in the last column. It appears that the differences are systematically dependent on spectral type, even though the large actual masses of the G giants have not been taken into account. The divergence of the early type stars is very apparent, however. The mass of a Bo giant as computed from the absolute magnitude is rather small, but even when the value 1·00 of Brill's curve is adopted, the difference is not larger than −0·10, which is impossible. Our curve does not in the least indicate the very large masses of the G giants.

In the second place we consider the numbers in Table IV. The arrangement of the table is the same as in the preceding case. The last column contains the differences between our curve and the dwarf one as given by A. Brill (*loc. cit.*). The run of these differences is much better than in the preceding case and only the B5, Ao, and A5 stars give values slightly larger than those of the other ones.

On the whole I think that Tables III. and IV. alone prove sufficiently that the spectroscopic binaries are dwarf stars. Yet I resolved to consider also the distances derived from Eddington's mass-luminosity relation. Therefore the values $\log \,.\, \overline{\sin^3 i}$ corresponding to the giants and dwarfs were separately determined. Table III. gives, from the spectral types A5-G5, the mean value $\log \,.\, \overline{\sin^3 i} = \bar{1}{\cdot}525$, which corresponds to a value of $\overline{\sin i} = 0{\cdot}695$. This value is about equal to the one usually adopted, viz., $\sin i = 0{\cdot}667$.

From Table IV. we obtain, when using all spectral types from O5 down to Ko, $\log \,.\, \overline{\sin^3 i} = \bar{1}{\cdot}905$, or $\sin i = 0{\cdot}927$, which I consider to be the correct value.

TABLE V.

Combined Values of the Corrections $\Delta m_1 = mag_{vis} - mag_{bol}$ *and* $\Delta m_2 = -2 \log . \frac{T_e}{5200}$.

Spectral Type.	Correction.	Spectral Type.	Correction.
Oe5	+1·26	Fo	−0·28
Bo	+0·79	F5	−0·20
B5	+0·28	Go	−0·03
Ao	−0·07	G5	+0·26
A5	−0·26	Ko	+0·53

3. *The Absolute Magnitudes of the Stars.*—The masses of the components of the different spectroscopic binaries will now be obtained by multiplying the observed values $m_1 \sin^3 i$ and $m_2 \sin^3 i$ by $\left(\frac{1}{0{\cdot}695}\right)^3$ and $\left(\frac{1}{0{\cdot}927}\right)^3$.

The masses obtained in this way are converted into absolute magnitudes by means of Eddington's well-known formula

$$L = -\tfrac{7}{2}\Delta \log M - \tfrac{15}{4} \log (1 - \beta) - 2\Delta \log T_e \quad . \quad (1)$$

Eddington tabulated the magnitudes corresponding to the different masses (table 14, *loc. cit.*). We must add to these values the term $\Delta m = -2 \log \frac{T_e}{5200}$ in order to obtain the magnitudes corresponding to the different spectral types. Moreover, the correction terms ($m_{vis} - m_{bol}$) have to be applied. The combined values of both corrections have been given in Table V. They were plotted against spectral type, and the corrections for the various spectral subdivisions are read from the curve drawn through these points.

The magnitudes which we obtain in the way described above are given in Table VI. On the left side of this table the absolute magnitudes obtained from the giant term are given; on the right the ones obtained from the dwarf term. In some cases the spectral type of the fainter component has not been observed. It has then been taken equal to that of the main star.

From the absolute magnitudes M_1 and M_2 of the two components we easily derive the absolute magnitude M_c of the system with the aid of the equation

$$M_c = -2{\cdot}5 \log (L_1 + L_2) \quad . \quad . \quad . \quad (2)$$

where $L_1 = 10^{-0{\cdot}4M_1}$ and $L_2 = 10^{-0{\cdot}4M_2}$ are the absolute luminosities of the two components.

The distance (r) of each binary system is thus obtained from the equation

$$M_c = m + 5 - 5 \log r, \quad . \quad . \quad . \quad (3)$$

in which m is the observed apparent magnitude of the system.

TABLE VI.

Absolute Magnitudes of Spectroscopic Binaries on the Supposition that they may be Giants or Dwarfs.

M_1 = mag. of main star; M_2 = mag. of component; M_c = combined magnitude.
r_1 = distance on giant hypothesis; r_2 = distance on dwarf hypothesis;
r_3 = distance obtained from trigonometrical parallax or group motion;
r_4 = distance from spectroscopic parallax (in parsecs).

No.	Spectra.	Giants.			Dwarfs.							Remarks.
		M_1.	M_2.	M_c.	M_1.	M_2.	M_c.	r_1.	r_2.	r_3.	r_4.	
1	O8 : O8	−5·76	− 5·72	−6·94	−4·10	− 3·95	− 5·23	2600	1860	3330	..	
2	B9 : –	+2·24	+ 4·74	+2·07	+2·24	+ 4·74	+ 2·07	120	120	200	..	TV Cas. var.
3	F8 : G1	+2·46	+ 2·04	+1·03	+5·86	+ 5·10	+ 4·27	69	12	20	..	
4	A5 : A5	−0·58	− 0·50	−1·74	+2·22	+ 2·22	+ 1·01	..	40	..	..	
5	B3 : B3	−0·05	+ 0·71	−0·91	+2·55	+ 3·79	+ 1·86	116	33	..	143	
6	F3 : F3	−0·04	+ 0·15	−1·16	+2·88	+ 3·16	+ 1·82	305	95	92	56	
7	Bo : –	+1·03	− 1·13	−1·56	+4·01	+ 1·16	+ 0·85	141	46	50	..	
8	Go : Go	+0·13	+ 0·13	−1·08	+3·19	+ 3·19	+ 1·59	180	53	..	62	
9	F2 : –	+0·59	+ 0·76	−0·52	+4·16	+ 4·40	+ 3·07	242	46	..	62	
10	Ao : –	+6·97	+ 2·69	+2·54	+6·97	+ 2·69	+ 2·54	..	..	..	..	RZ Cas. var.
11	B8 : –	−0·61	+ 8·13	−0·61	−0·61	+ 8·13	− 0·61	40	40	33	..	β Per. var.
12	O8 : O8	−4·91	− 3·60	−5·58	−3·33	− 1·89	− 3·96	3900	1870	2500	..	
13	B8 : B8	−2·05	− 2·18	−2·90	+0·18	+ 0·03	− 1·10	..	120	..	..	
14	B3 : –	+1·63	+ 5·21	+1·42	+1·63	+ 5·21	+ 1·42	38	38	200	..	λ Tau. var.
15	B3 : B3	−2·98	− 2·65	−4·07	−0·93	− 0·57	− 1·96	..	..	..	..	
16	A2 : A2	+0·56	+ 6·28	+0·50	+4·24	+ 9·88	+ 4·17	..	1	..	..	
17	B9 : –	+2·56	+ 2·64	+1·40	+6·18	+ 6·18	+ 4·97	..	0·5	..	..	
18	A3 : F	−1·12	− 0·44	−2·02	−0·04	+ 2·45	− 0·40	..	57	..	..	
19	B9 : B9	−2·13	− 1·56	−3·07	+0·04	+ 0·78	− 0·84	..	161	..	..	
20	Go : Go	−0·32	+ 0·21	−1·28	−0·32	+ 0·21	− 1·28	20	20	16	..	Visual double.
21	A1 : A1	−1·04	− 0·76	−2·11	+1·62	+ 1·96	+ 0·58	360	107	..	90	
22	B1 : B1	−4·34	− 4·21	−5·48	−2·65	− 2·53	− 3·80	609	281	..	143	
23	B2 : B2	−3·07	− 2·50	−4·02	−1·11	− 0·42	− 1·90	544	205	142	125	
24	Ao : A5	+2·01	+ 3·02	+1·46	+5·61	+ 7·02	+ 4·84	..	..	1	..	
25	Ao : Ao	−1·63	− 1·52	−2·78	+0·71	+ 0·71	− 0·50	97	33	26	..	
26	A5 : A5	+0·50	+ 1·26	−0·36	+3·90	+ 4·54	+ 2·99	..	26	..	..	
27	A2 : –	−3·04	− 0·21	−3·35	−0·92	+ 2·76	− 1·12	234	77	27	..	
28	A3 : –	−0·52	+ 0·04	−1·47	+2·28	+ 3·02	+ 1·41	..	55	..	..	
29	Ao : –	−1·63	− 1·27	−2·67	+0·69	+ 1·17	− 0·29	530	171	75	..	
30	Oe5 : Oe5	−6·40	− 6·27	−7·54	−5·42	− 6·14	− 7·02	..	4130	..	..	

TABLE VI.—*continued.*

No.	Spectra.	Giants.			Dwarfs.							Remarks.
		M_1.	M_2.	M_C.	M_1.	M_2.	M_C.	r_1.	r_2.	r_3.	r_4.	
31	Ao : Fo	+1·21	+ 4·60	+0·98	+4·57	+ 7·52	+ 4·28	6	1	3	..	
32	F5 : F5	+0·19	+ 0·84	−0·72	+3·32	+ 4·20	+ 2·50	192	44	..	38	
33	M1 : M1	+4·68	+ 6·80	+4·07	+8·1 :	+ 8·4 :	+ 7·04	4	1	13	..	
34	Ao : –	+0·25	+ 0·25	−0·95	+0·25	+ 0·25	− 0·95	58	58	13	..	Castor. $m_1=m_2$ was adopted.
35	F3 : F3	−0·88	− 0·60	−1·95	+1·84	+ 2·32	+ 0·46	389	128	..	38	
36	F8 : –	+1·74	+ 4·98	+1·49	+5·30	+ 8·98	+ 5·10	63	22	19	..	
37	F8 : –	+2·33	+ 2·68	+1·06	+2·33	+ 2·68	+ 1·06	30	30	40	..	ε Hya. var.
38	A2 : –	−0·68	− 0·32	−1·71	+2·00	+ 3·48	+ 1·14	177	82	..	84	
39	Fo : Fo	+4·60	+ 8·24	+4·40	+4·60	+ 8·24	+ 4·40	24	24	63	..	S Ant. var.
40	F5 : Ao	−0·44	+ 0·17	−1·37	+2·42	+ 3·13	+ 1·54	106	28	38	71	
41	F8 : F8	+5·74	+ 7·51	+4·90	+5·74	+ 7·51	+ 4·90	40	40	44	..	W UMa. var.
42	F2 : –	+5·46	+ 5·94	+4·48	+9·02	+10·02	+ 8·24	8	1	..	28	
43	Ao : Ao	−2·75	+ 2·25	−2·85	−0·63	+ 5·89	− 0·67	..	126	..	..	
44	A2 : F	+9·68	+10·66	+8·89	+12 : ?	+14 : ?	+11 : ?	2	1	77	63	
45	Ao : Ao	+0·17	+ 1·29	−0·56	+3·65	+ 4·73	+ 3·10	..	15	..	..	
46	G5 : –	+1·50	+ 2·02	+0·54	+4·96	+ 5·46	+ 3·98	347	69	29	36	
47	A5 : –	−1·91	+ 4·94	−1·99	+0·24	+ 8·46	+ 0·23	279	100	24	14	
48	Ao : Ao	−1·95	− 1·51	−2·95	+0·37	+ 0·87	− 0·60	650	219	..	125	
49	F8 : Ko	−0·34	+ 0·29	−0·60	−0·34	+ 0·29	− 0·60	490	490	..	125	RS CVn. var.
50	B2 : B2	−4·08	− 3·16	−4·89	−2·39	− 1·23	− 3·11	166	73	..	27	
51	A2 : A2	−1·08	− 0·96	−2·22	+1·52	+ 1·68	+ 0·40	53	25	27	19	
52	F5 : F5	−0·52	− 0·44	−1·69	+2·28	+ 2·44	+ 1·16	200	54	..	30	
53	G4 : K4	+0·36	+ 1·66	−0·32	+3·40	+ 4·76	+ 2·74	7	2·6	1·3	..	
54	F5 : F5	−0·36	+ 0·24	−1·29	+2·66	+ 3·44	+ 1·80	417	100	..	38	
55	Ao : –	+5·21	+ 6·61	+4·57	+5·21	+ 6·61	+ 4·57	13	13	77	..	Boss 3825 var.
56	B3 : B3	+0·34	+ 3·11	−0·08	+0·34	+ 3·11	− 0·08	394	394	333	..	U CrB. var.
57	B8 : B8	−5·24	− 5·24	−6·44	−3·57	− 3·57	− 4·77	2000	912	..	111	
58	B1 : B1	−4·61	− 3·75	−5·44	−2·89	− 2·00	− 3·70	466	209	125	143	
59	F8 : F8	+0·22	+ 0·60	−0·80	+3·30	+ 3·90	+ 2·38	300	46	..	30	
60	Ao : Ao	−1·63	− 0·39	−2·32	+0·70	+ 2·41	+ 0·15	..	167	..	..	
61	Go : –	+2·15	+ 5·57	+1·52	+5·63	+ 9·49	+ 5·46	18	3	10	..	
62	F2 : F2	−0·02	+ 0·22	−1·10	+2·94	+ 3·50	+ 2·00	..	66	..	..	
63	Ao : Ao	−0·83	+ 0·41	−1·52	+1·85	+ 3·69	+ 1·33	123	33	32	..	
64	B3 : B3	−0·57	− 0·40	−2·00	−0·57	− 0·40	− 2·00	398	398	200	..	U Oph. var.
65	B3 : B3	−1·39	+ 1·11	−1·76	−1·39	+ 1·11	− 1·76	187	187	167	..	u Her. var.

TABLE VI.—*continued.*

No.	Spectra.	Giants. M_1.	Giants. M_2.	Giants. M_C.	Dwarfs. M_1.	Dwarfs. M_2.	Dwarfs. M_C.	r_1.	r_2.	r_3.	r_4.	Remarks.
66	A2 : A2	+1·69	+ 2·17	+0·71	+1·69	+ 2·17	+ 0·71	203	203	250	..	TX Her. va
67	Fo : –	+0·87	+ 0·84	−0·34	+3·96	+ 4·24	+ 2·90	89	22	..	75	
68	F2 : F2	+2·47	+ 3·39	+1·67	+2·47	+ 3·39	+ 1·67	124	124	91	..	Z Her. var.
69	Ko : K_5	+2·34	+ 5·30	+2·05	+5·91	+ 9·14	+ 5·66	28	5	5	..	
70	F5 : F5	+3·08	+ 3·56	+2·10	+6·88	+ 7·32	+ 5·92	65	12	..	46	
71	Fo : Fo	+0·16	+ 0·20	−1·00	+3·24	+ 3·48	+ 2·15	304	71	..	112	
72	A3 : A3	−1·12	− 0·16	−1·50	+1·40	+ 2·84	+ 0·77	256	90	..	63	
73	Ao : A3	+4·80	+ 4·92	+3·66	+4·80	+ 4·92	+ 3·66	..	..	..	..	RX Her. var.
74	F2 : A5	−0·78	− 0·80	−1·59	+1·90	+ 1·90	+ 0·70	568	198	..	200	
75	B8 : B5	−2·39	− 3·81	−4·44	−2·39	− 3·81	− 4·44	377	377	333	..	β Lyræ.
76	Ao : Ao	+0·65	+ 0·78	−0·49	+3·85	+ 4·13	+ 2·78	..	33	..	..	
77	F5 : F5	−0·36	− 0·36	−1·56	+2·58	+ 2·58	+ 1·38	..	104	..	..	
78	B8 : B9	−0·96	+ 2·48	−1·19	−0·96	+ 2·48	− 1·19	522	522	333	..	RS Vul. var
79	F2 : B8	..	..	..	−8·14	− 6·19	− 8·51	..	4100	..	..	v Sgr. (see Table I.).
80	B3 : B3	−0·49	+ 1·84	−0·89	−0·49	+ 1·84	− 0·89	435	435	500	..	Z Vul. var.
81	Ao : –	+0·69	+ 1·89	−0·01	+4·09	+ 5·53	+ 3·45	162	33	166	71	
82	B8 : B8	−3·48	− 3·09	−4·96	−1·49	− 0·95	− 2·44	1000	333	167	..	
83	F5 : F5	−1·36	− 1·32	−2·54	+1·32	+ 1·25	+ 0·08	479	143	..	92	
84	O9 : O9	−4·35	− 3·97	−5·38	−2·83	− 2·69	− 3·97	3160	1650	1439	..	
85	Ao : –	+2·77	+ 3·89	+2·04	+6·41	+ 7·85	+ 5·78	46	..	67	..	
86	A2 : –	−1·76	− 1·56	−2·87	+0·48	+ 0·89	− 0·54	463	160	..	77	
87	B2 : B2	−3·07	− 1·87	−3·77	−3·07	− 1·87	− 3·77	1490	1490	1000	..	
88	B3 : B3	−0·56	− 0·38	−1·67	+1·99	+ 2·23	+ 0·90	169	57	..	125	
89	F3 : F3	+0·32	+ 1·11	−0·53	+3·60	+ 4·56	+ 2·81	238	51	..	100	
90	Ao : Ao	+0·57	+ 0·57	−0·73	+3·85	+ 3·85	+ 2·65	..	43	..	..	
91	A9 : A9	−0·25	− 0·21	−1·43	+2·63	+ 2·63	+ 1·43	463	124	..	111	
92	F3 : F5	−4·88	− 3·01	−5·39	−3·24	− 0·84	− 3·63	..	..	..	30	Not known which comp. of vis. double is spec. bin.
93	B5 : B5	+1·03	+ 1·97	+0·23	+4·56	+ 5·48	+ 3·76	77	15	28	..	
94	Bo : Bo	−4·61	− 4·37	−5·69	−2·97	− 2·89	− 4·13	..	1560	..	..	
95	B3 : B3	−1·73	− 1·18	−2·68	+0·51	+ 1·19	− 0·39	620	216	..	333	
96	Bo : –	−5·05	− 4·24	−5·90	−3·63	− 2·78	− 4·44	2180	1250	..	500	
97	F5 : F5	−1·48	− 1·41	−2·65	+1·42	+ 1·56	+ 0·28	445	121	..	53	

In this way we obtain two values for the distance of each system, the first from the giant mass, which will be the larger one, the second corresponding to the dwarf mass. The last four columns of Table VI. contain (1) the distance obtained from the giant mass, (2) the distance from the dwarf mass, (3) that computed from trigonometrical parallax or group motion, and (4) the distance computed from the spectroscopic parallax. The values for the last named were taken from Beer's catalogue (*loc. cit.*).

4. *Comparison of the Observed Distances with the Computed Ones.*—From Table VI. we derive the differences $\log r_{obs} - \log r_{comp}$. To begin with, these were plotted against spectral type. Figure 2 (*a*) contains the differences obtained from the giant masses while 2 (*b*) gives those obtained from the dwarf masses. Crosses indicate the Algol or eclipsing variables which are not influenced by the value of $\log . \overline{\sin^3 i}$, open circles give the differences obtained from trigonometrical parallaxes, and finally black dots are the differences derived from spectroscopic data.

One glance at fig. 2 (*a*) shows that the differences computed from the giant masses are nearly all negative. For the trigonometrical parallaxes the mean of the differences is − 0·234, while for the spectroscopic ones it is − 0·436. The mean of all stars together is − 0·336. This indicates that the absolute magnitudes derived from the giant term are too small (1·15 and 2·18 magnitudes respectively).

If, on the other hand, we consider fig. 2 (*b*), in which the differences obtained from the dwarf masses have been plotted, we see that positive and negative differences are equally frequent over the whole range of spectral types. The mean difference from the trigonometrical parallaxes in this case is + 0·126, the mean from the spectroscopic ones is − 0·018, while for all stars together we find + 0·062.

We thus obtain a second proof that the spectroscopic binaries are dwarfs. The values of $\log r_{obs} - \log r_{comp}$ are not systematically dependent on spectral type. This proves that the mean value $\overline{\sin i}$ is the same for all spectra.

5. *The Relation between Mass and Parallax.*—It may be useful to consider more closely the distances derived from the masses. As a rule we may expect that the computed distances will be systematically dependent on the masses. When for $\overline{\sin i}$ the mean values 0·695 and 0·927 respectively are adopted, the possibility remains that the individual values of $\sin i$ are scattered round the mean.

If we consider, for instance, the case when the actual value of $\sin i$ is larger than the mean, then the mass which is obtained after multiplication by $\left(\frac{1}{0{\cdot}695}\right)^3$ or $\left(\frac{1}{0{\cdot}927}\right)^3$ will be too large, and the absolute magnitude too small. Evidently the computed distance is too large and the difference $\log r_{obs} - \log r_{comp}$ will be negative, while, on the other hand, this difference will be positive if $\sin i$ is smaller than the mean. Large masses may either be real or be caused by large positive errors, and from the latter we expect the differences, $\log r_{obs} - \log r_{comp}$, to be systematically dependent on mass. With small masses positive excesses must

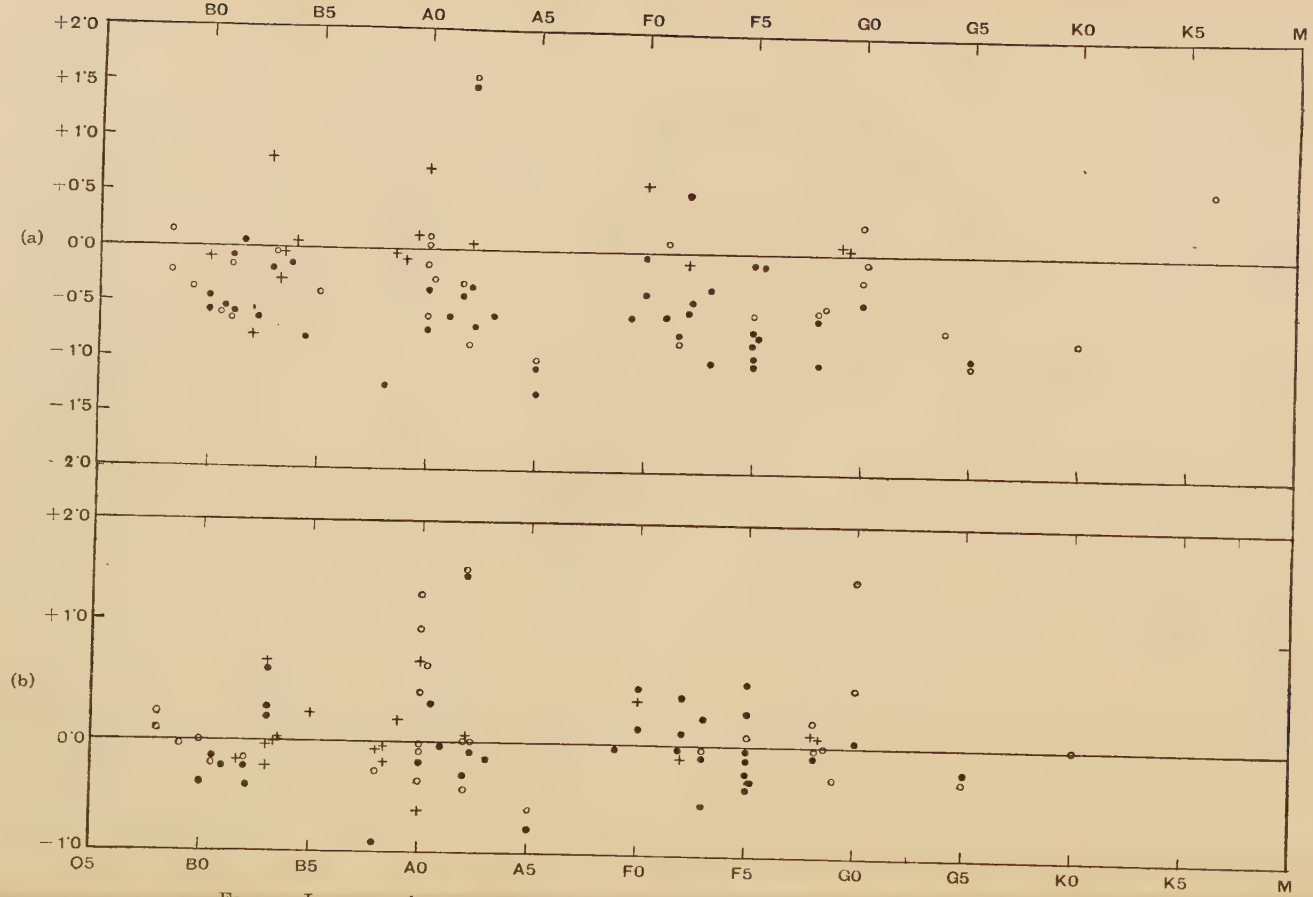

FIG. 2.—Log r_{obs} − log r_{comp} plotted against spectral type. (*a*) Giant term. (*b*) D[illegible]

occur; for the larger masses, negative ones. This occurs if either the correct or the erroneous value of $\overline{\sin i}$ is used, but in the latter case matters will be complicated as the relation between logarithm of mass and spectral type is not a linear one.

In the figures 3 (*a*) and 3 (*b*) the values $\log r_{obs} - \log r_{comp}$ are plotted against the logarithm of the mass. Just as in the preceding case, open circles indicate evidence obtained from trigonometrical data, black dots that derived from spectroscopic data. The Algol variables have been omitted, as for these stars the masses m_1 and m_2 are directly given.

If, again, fig. 3 (*a*) is considered first, and if a linear relation between the differences $\log r_{obs} - \log r_{comp}$ is adopted, we obtain from the trigonometrical data the equation

$$\log r_{obs} - \log r_{comp} = -0{\cdot}062 - 0{\cdot}476 \log (\text{mass}) \pm 0{\cdot}458 \text{ (mean error)} \quad (4)$$

and from the spectroscopic ones,

$$\log r_{obs} - \log r_{comp} = +0{\cdot}071 - 0{\cdot}718 \log (\text{mass}) \pm 0{\cdot}457 \text{ (mean error)} \quad (5)$$

The curve (4) has been indicated in the figure by a full drawn line, and curve (5) by a dotted one.

From the dots in fig. 3 (*b*) we get from trigonometrical and spectroscopic data the equations (6) and (7), which are indicated in the same way.

$$\log r_{obs} - \log r_{comp} = +0{\cdot}344 - 0{\cdot}580 \log M \pm 0{\cdot}505 \text{ (mean error)} \quad (6)$$

$$\log r_{obs} - \log r_{comp} = +0{\cdot}213 - 0{\cdot}602 \log M \pm 0{\cdot}320 \text{ (mean error)} \quad (7)$$

With the smaller masses the differences increase, while with the larger ones they decrease. This was anticipated.

We have adopted as the real value of $\overline{\sin i}$, 0·927. It is evident that it is impossible for the computed masses to be very much too large. On the other hand, values of $\sin i$ may occur which are much smaller than 0·927, and as $\overline{\sin i}$ enters with the third power into our computations, we expect the computed masses to be very much too small in these instances. If this is the case, the absolute magnitudes derived from them will be too large also, and the distances too small. In other words, we may expect that a few very large positive excesses will be found while no compensating negative ones will occur. The distribution of the differences $\log r_{obs} - \log r_{comp}$ in Table VII. proves that our surmise is correct. It may seem possible to derive from the numbers in Table VII. the actual distribution of the values $\sin i$ round their mean. I do not think that any reliable results will be obtained, as the differences in Table VII. will be largely influenced by the probable errors in the determination of parallax and magnitude and by the remaining uncertainties in the determination of the functions $m_1 \sin^3 i$, $m_2 \sin^3 i$, and $\overline{\sin i}$. In any case a separate treatment of the trigonometrical and spectroscopic data would be necessary, so that the numbers would be rather small for an exact determination.

As to the differences $\log r_{obs} - \log r_{comp}$ it should finally be remarked that apparently from formulæ (4), (5), (6), and (7) the spectroscopic

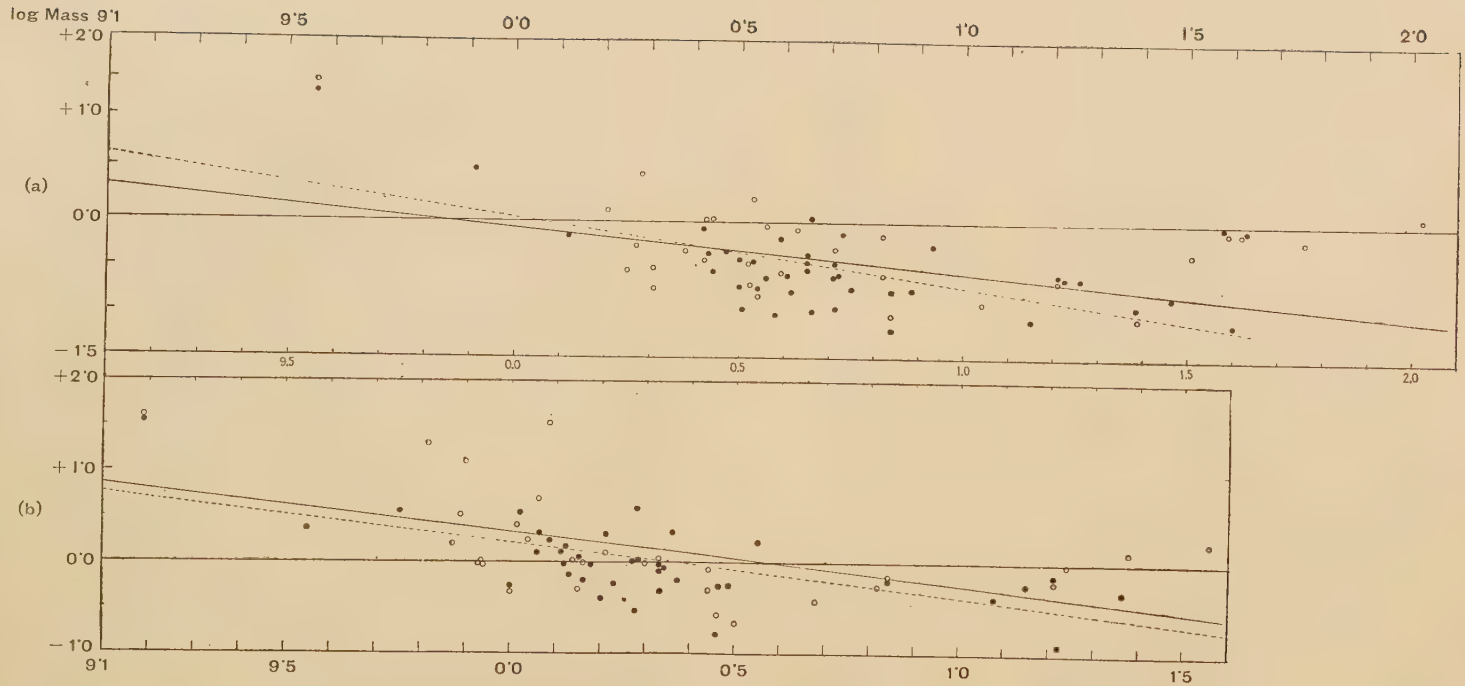

FIG. 3.—Log r_{obs} − log r_{comp} plotted against log of mass. (*a*) Giant term. (*b*) Dwarf term.

parallaxes are systematically larger than the trigonometrical ones. This agrees with the results of van Rhijn,* who finds that for the apparently brighter stars, the spectroscopic parallaxes are too large.

6. *The Relation of Absolute Magnitude to Spectral Type.*—In the figure 4*a* I have plotted the absolute magnitudes obtained from the giant hypothesis against spectral type. Algol or eclipsing variables are indicated by open circles; all other stars by black dots. Two circles or dots indicating the components of one and the same system have been connected. It appears that the diagram which we obtain in this way is a close imitation of the Russell-Hertzsprung diagram as obtained by P. Doig,† F. C. Leonard,‡ and G. Shajn § for the visual double stars. The giant curve and the dwarf one, though having rather improbable inclinations, are well indicated. I think, however, that by using the term $\overline{\sin i} = 0{\cdot}695$ we convert many dwarf stars into giants. It is easily proved that this is really the case. Therefore we consider the stars for which the trigonometrical or spectroscopic parallaxes are given. From the catalogue by A. Beer the absolute magnitudes M_b of these stars are obtained. The correct absolute magnitudes of the components are found by applying to the magnitudes in columns M_1 and M_2 (giants) of Table VI., the corrections $M_b - M_c$, and, again, the corrected magnitudes are plotted against spectral type. Figure 4*b* is thus obtained, in which is indicated the main sequence of dwarfs only.

If we plot the dwarf magnitudes M_1 and M_2 of Table VI. against spectral type the issue is figure 4*c*. Comparing 4*b* and 4*c* we obtain final evidence that the spectroscopic binaries are indeed genuine dwarfs. In fig. 4*c* the main sequence of dwarfs may be continued from the very early types down to the K and M stars. There is no single indication of the giant curve as was the case in 4*a*. There is only one outstanding case, viz., the star υ Sagittarii already noted in Table I. (No. 79), which appears in our figure as a super-giant. The mass function of this star is, however, but imperfectly known. Having thus obtained three definite proofs that the spectroscopic binaries are dwarf stars with small masses, we adopt as the correct value of $\overline{\sin i}$ the one obtained from Table IV., viz., $\overline{\sin i} = 0{\cdot}927$. In the following only this value will be taken into account.

7. *The Masses of the Stars with Different Spectral Types.*—In Table VIII., column 2, I have given the masses of the dwarfs of different spectra as obtained from our curve after applying the term $\left(\frac{1}{0{\cdot}927}\right)^3$ to the observed values of $m_1 \sin^3 i$ and $m_2 \sin^3 i$. They are compared in this table with the corresponding masses obtained by various authors from different groups of stars. The survey, however, is by no means intended to be complete. Column 3 contains the masses of the visual binaries given by A. Brill.‖ These same values have already been used in Table IV. Column 4 gives the masses derived by W. Rabe.¶

* *Gron. Publ.*, 34 and 37.
† *M.N.*, **82**, 372, 1921.
‡ *L.O.B.*, X. No. 343, 169, 1923.
§ *A.N.*, 5404, 49, 1925.
‖ *Loc. cit.*
¶ *A.N.*, 5525.

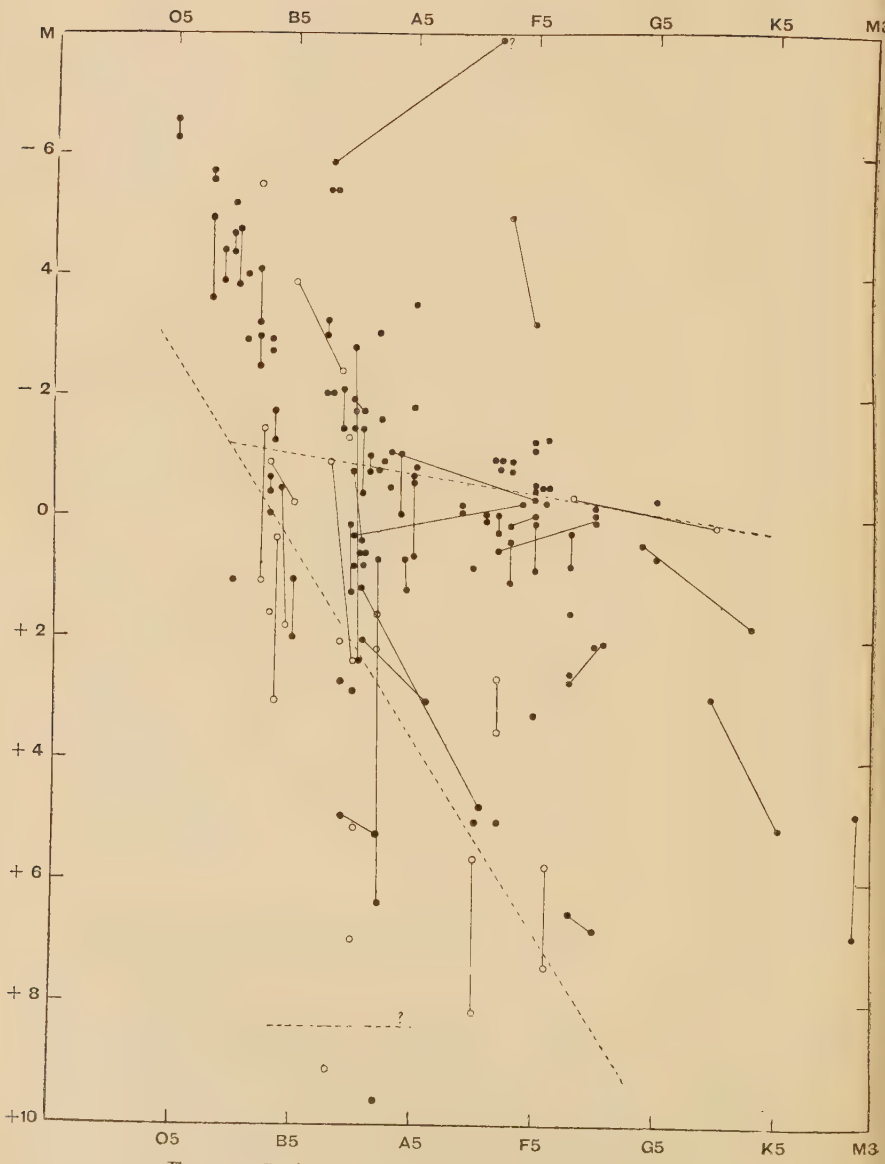

Fig. 4*a*.—Hertzsprung-Russell diagram from giant term.

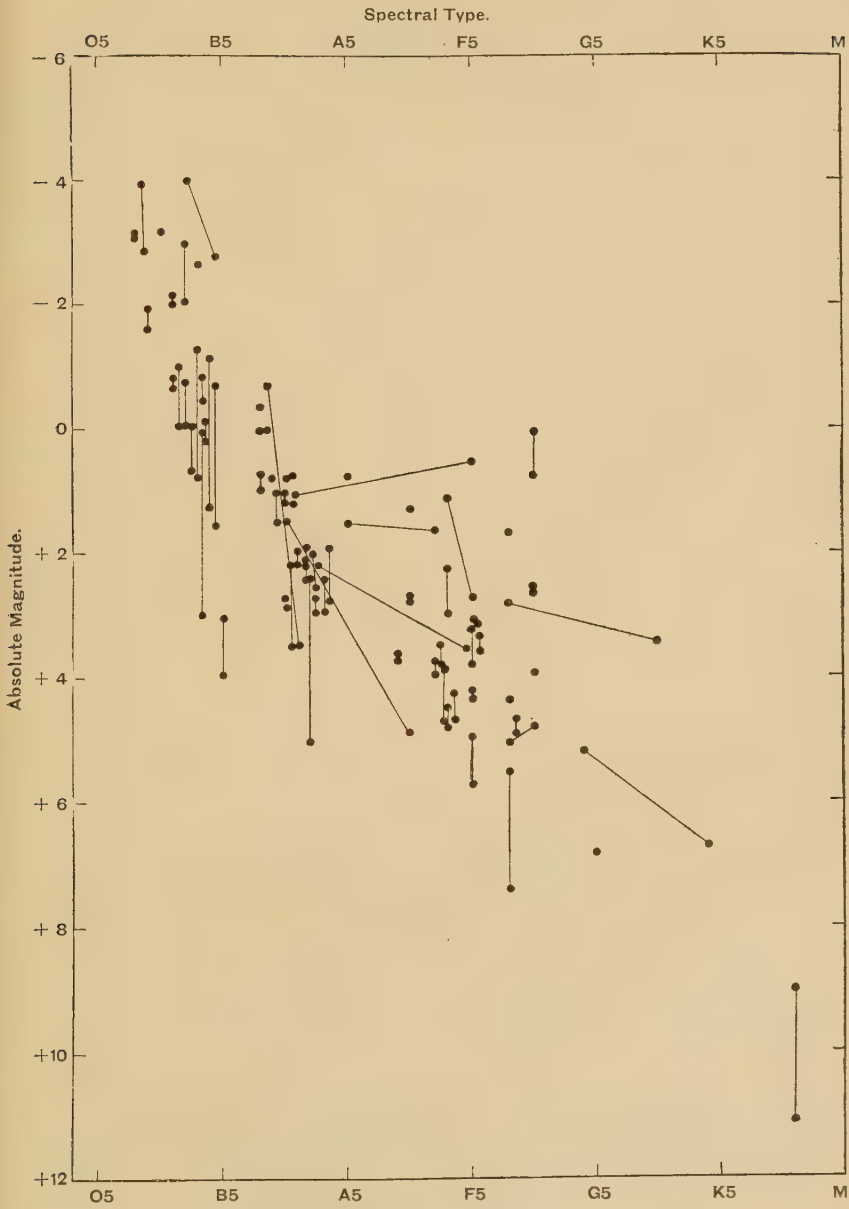

FIG. 4*b*.—Hertzsprung-Russell diagram for spectroscopic binaries of known parallax.

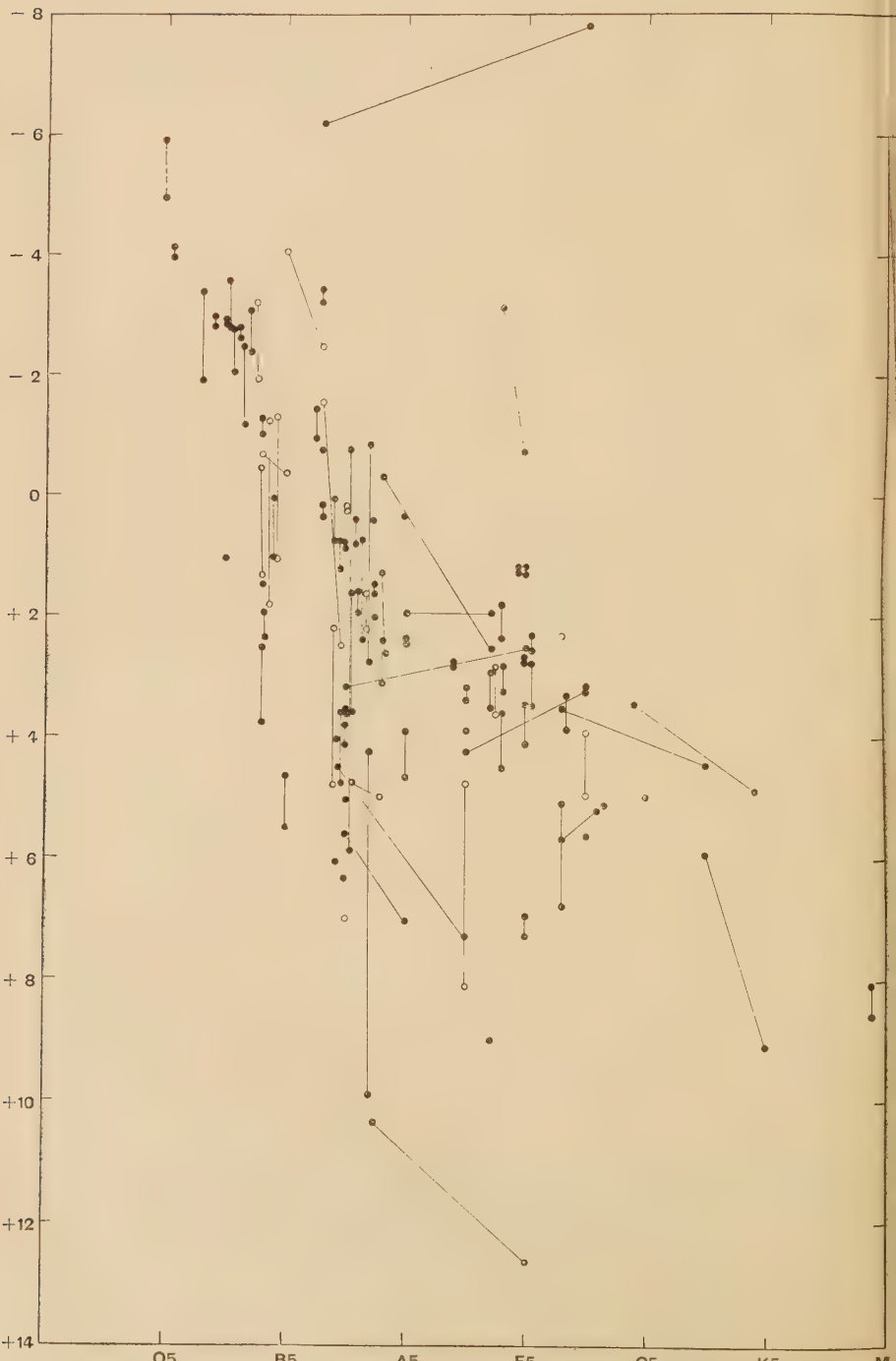

FIG. 4c.—Hertzsprung-Russell diagram from dwarf term.

TABLE VII.

Distribution of the Differences $\log r_{obs} - \log r_{comp}$ *from Trigonometrical* (Δ_{tr}) *and Spectroscopic* (Δ_{sp}) *data.*

$\Delta(\log r_{obs} - \log r_{comp})$.	$>+1{\cdot}0$.	$+1{\cdot}0$.	$+0{\cdot}8$.	$+0{\cdot}6$.	$+0{\cdot}4$.	$+0{\cdot}2$.	$0{\cdot}0$.	$-0{\cdot}2$.	$-0{\cdot}4$.	$-0{\cdot}6$.	$-0{\cdot}8$.	$-1{\cdot}0$.	$<-1{\cdot}0$.
Numbers Δ_{tr}	4	0	1	2	3	6	7	5	2	1	0	0	
Numbers Δ_{sp}	1	0	1	3	4	6	8	9	2	0	2	0	

TABLE VIII.

Masses of Dwarf Stars of Different Spectral Types.
(Unit = ☉).

Spectr.	Kreiken.	A. Brill.	W. Rabe.	Von Zeipel Lindgren.	Wallenquist.	Seares.	Probable Mean.
O_5	100	100	..	..	..	..	100
Bo	16	10	15·7	..	..	..	14
B_5	3·5	4·5	7·2	..	..	..	4·5
Ao	2·1	2·8	3·8	..	..	6·0	2·8
A_5	1·6	2·0	2·2	..	..	4·0	1·9
Fo	1·4	1·5	1·8	..	..	2·5	1·5
F_5	1·2	1·2	1·4	1·88	1·88	1·5	1·2
Go	1·1	1·0	1·1	..	..	1·0	1·0
G_5	1·0	0·8	0·94	1·01	..	0·8	0·9
Ko	0·9	0·7	0·75	..	..	0·7	0·75

On some Early Photographs of the Planets by Light of Different Colours.

By W. H. Wright.

In the George Darwin Lecture for 1928,* which had for its subject the photography of the planets by light of different colours, I stated, or rather implied, that the first observations of the planets with selective filters undertaken for analytical purposes were made in 1912 by Professor R. W. Wood. My attention has since kindly been called by members of the staff of the Lowell Observatory, through the intermediacy of Professor Russell, to two excellent sets of observations of this kind that were made respectively by MM. A. de la Baume Pluvinel and F. Baldet † in France, and by G. Tikhoff ‡ in Russia, on the approach of Mars in 1909.

Pluvinel and Baldet, observing from Pic du Midi, appear to have been especially interested in photographing details on the Martian surface, and when they found ordinary violet sensitive plates did not reveal these features they discarded them in favour of plates sensitive to the

* *M.N.*, **88**, 709, 1928.
† *C.R.*, **149**, 838, 1909.
‡ *Pulk. Mitt.*, **4**, No. 42, 1911; see also *Obs.*, **33**, 41, 1910.

less refrangible region of the visible spectrum, supplemented by a yellow colour-screen, which met their requirements. They described some of the differences between violet and orange photographs, namely, the complete absence from the former of the dark areas ordinarily seen in the telescope, and the relatively great strength of the polar caps. At the southern, or summer pole, the cap was small and bright, while the northern rim was marked by a large pale area. Limb light was pronounced in the pictures taken by violet light.

Tikhoff at Pulkova, photographing by red, orange, and green light, found a progressive weakening of the planet's dark markings with increasing refrangibility of the light used, and a simultaneous strengthening of the polar caps and limb light. Fading of the markings and the development of limb light he attributed to the action of the Martian atmosphere, suggesting that the effect would be still more pronounced in blue light. From this suggestion we may conclude that he was unfamiliar with the paper of Pluvinel and Baldet, published a year or more earlier, in which the surmise was verified. Tikhoff explained the observed chromatic characteristics of the south polar cap without recourse to the planet's atmosphere; its weakness in red light and strength in green led him to believe that its colour is not white but greenish, and he suggested that the cap is composed of green ice.

In the same paper Tikhoff discussed photographs of Saturn taken in different colours, and recorded several of the phenomena later described by Wood. Finally, he expressed the opinion that observations of this kind provide a valuable means for studying the constitution of the planets.

The observations at Pic du Midi and Pulkova, taken together, establish some of the principal facts that have been brought to attention by more recent observers. That the suggestions they afforded were not taken advantage of is difficult to explain on any other ground than that these early observations were overlooked or forgotten. Wood, who seems generally to have been regarded as the pioneer observer in this field, certainly knew nothing of them, for they anticipate some of his more important findings respecting Saturn; and it can fairly be inferred from literature of the period since his publication that they have not generally been known to students of planetary phenomena.

It may be appropriate to enumerate a number of phenomena which, in addition to those described above, have been detected in the more recent observations of Mars, especially as one of them bears on Tikhoff's interpretation of the polar cap. These are: the darkening of the limb in photographs taken by light of longer wave-lengths, especially near opposition; the larger size of the planet in short-wave photographs; the occurrence of clouds of two types in the Martian atmosphere; and the relatively high content of violet in the light of the southern cap when it is close to the limb. Tikhoff failed to detect limb-darkening, and as a consequence excluded atmospheric absorption as an explanation of the faintness of the polar cap in his red photographs. It may be remarked that he did not observe under the best conditions for the detection of limb-darkening, because Mars was at considerable

phase (22° to 37°) at the time at which he worked, and the limb was favoured by illumination from a high sun.

Tikhoff and I agree in assigning to atmospheric obscuration the progressive fading of the dark Martian areas with decreasing wave-length of the light used in observing, but beyond this I am not aware that the general interpretations of the colour photographs of Mars, that I have ventured to suggest, trespass on those of earlier workers.

I regret that my quite inexcusable ignorance of the important papers of MM. Pluvinel and Baldet, and of M. Tikhoff, prevented my reference to them in the George Darwin Lecture, but my natural discomfiture at having overlooked them is in part allayed by the pleasure the present note affords in permitting a more complete reference to them than would have been possible on that occasion.

The Longitude of the Royal Observatory, Cape of Good Hope, from Wireless Signals, October–November 1926. By H. Horrocks, M.A.

(*Communicated by H.M. Astronomer.*)

Wireless Observations and Apparatus.—The observations of the Bordeaux signals (8^h 1^m–6^m) and Annapolis signals (10^h 10^m–15^m) at the Cape Observatory during the period of the *International Longitude Programme* were made by Dr. Halm with a four-valve receiving set (two H.F. amplifiers and two L.F. amplifiers). The apparatus, designed by Halm, by means of which the signal times were recorded, consists of a gramophone motor, the rotating table of which completes an electric circuit operating a chronograph once per revolution. The position of the electric contact along the circular path of the contact-maker can be varied by the observer so as to make the beats of the Fuess chronograph synchronise with the signals to which he listens, the motor having been so regulated that the period of revolution agrees very closely with that of the signals. The second pricker of the chronograph records the beats of the standard clock.

The chronograph was brought into operation towards the end of each minute, and the record corresponding to the commencement of the dash was measured, each wireless time-determination thus depending on five signals.

Clock Error.—This was derived from the observations of clock stars in the current observing programme with the reversible transit-circle, the determinations being smoothed graphically. Eichelberger's star places were used. Observations were secured on 38 nights during the above programme, ten clock stars being observed whenever possible, though on several occasions only three or four observations could be obtained. The clock in use as standard was Riefler 150.

Personality in Wireless Observations.—No correction can be applied on this account. Reference will be made later to a series of simul-

taneous observations by different observers, taken in 1928, which indicate an error of this nature of the order of a few hundredths of a second.

Derivation of Longitude.—Observations of Bordeaux, Annapolis, and Eiffel Tower signals (B.A.E.) at the Cape, Greenwich, Paris, and Washington (C.G.P.W.) have been employed, the mean of the times of signals 1 and 306 of each series for each day being regarded as a "single observation" for the present discussion. For a few days, times for one only of the signals 1 and 306 have been published. In the formation of the means over the whole period of the time-differences at two observatories derived from a single series of signals, it was found to be immaterial whether full or half weight was assigned in these cases. As error of observation of the signals does not outweigh all other contributory causes to the deviations from the mean, time-differences derived from the mean of signals 1 and 306, or from one only of these, are given equal weight in weighting the means.

For each pair of observatories the deviations of the daily time-differences from the mean were formed for each set of signals, and the mean deviations computed for each set separately. It appeared that for any given pair of observatories the mean deviations derived from different sets of signals are substantially equal. In fact, the only discordance exceeding $0^s{\cdot}001$ is one of $0^s{\cdot}004$ arising from the Cape–Paris differences for Bordeaux and Annapolis signals. Differences at any one observatory between the accidental errors of the observations of different sets of signals are accordingly considered negligible in assigning weights, on the basis of the internal agreement of the daily values, to the means given in column 4 of Table I. A single value for the mean deviation, entered in column 6, was therefore assigned to each pair of observatories.

If the deviations followed the law of accidental errors we should now expect to be able to express the entries of column 6 in terms of four mean errors, η_G, η_P, η_W, η_C, in the forms $\sqrt{(\eta_G^2 + \eta_C^2)}$, etc. As may be expected from the apparently systematic variation in the daily time-differences, the discordances are very large. A solution for η_G, etc., was however made, giving

η_G.	η_P.	η_W.	η_C.
·0101	·0134	·0144	·0390

account being taken of the number of observations on which the entries of column 6 rested. The mean deviations adopted for the purpose of weighting the observations (column 7) were based on this solution. Columns 8 and 9 give the adopted weights of the means and of the single observations.

Results.—The deduced longitudes are

	h	m	s		s
Cape (old transit-circle)	1	13	54·596	E	± 0·0066
Paris (Cassini's meridian)	0	9	20·909	E	± 0·0026
Washington	5	8	15·752	W	± 0·0027

These determinations are in satisfactory agreement with other

published values derived from observations of wireless signals,* and the Cape longitude is in good agreement with Christie's determination by telegraphic signals in 1908.†

‡ Table I.

Observatories.	Signals.	No. Obs.	Mean Time Difference. Oct. and Nov.	Correction for Time of Transmission.§	Mean Deviation. Obs.	Mean Deviation. Adopted.	Adopted Wt. of Mean.	Wt. of Single Observation.
	h m		h m s		s	s		
C–G	A 10 10	28	1 13 54·662	− ·61τ	0·050	0·040	5	0·18
P–G	E 9 31	51	0 9 21·011	+ ·03τ	0·018	0·017	50	1·00
P–G	E 22 31	58	21·011	+ ·03τ			60	1·00
G–W	B 20 1	51	5 8 15·728	+ ·48τ	0·015	0·018	45	0·89
G–W	A 20 10	49	15·779	− ·52τ			45	0·89
C–P	A 10 10	32	1 4 33·628	− ·59τ	0·034	0·041	5½	0·17
C–P	B 8 1	57	33·615	− ·75τ			10	0·17
P–W	B 20 1	55	5 17 36·741	+ ·50τ	0·021	0·020	40	0·72
P–W	A 20 10	47	36·798	− ·55τ			35	0·72
C–W	B 8 1	48	6 22 10·332	− ·26τ	0·048	0·042	7½	0·16
C–W	A 10 10	36	10·425	−1·13τ			6	0·16

The time τ for the transmission of the signals per 100° arc of a great circle, which was included as an unknown in the equations, was determined to be $0^s{\cdot}055 \pm 0^s{\cdot}0048$. Assuming transmission along the earth's surface, this gives the velocity in m.p.s. as 125,000 ± 11,000.

It may be remarked that the probable error of a time-difference of unit weight (single determination, Paris–Greenwich) derived from the solution of the normal equations is $\pm 0^s{\cdot}032$, which is considerably larger than the value $\pm 0^s{\cdot}014$ which would be inferred from the mean deviations from the means.

This discordance appears to be due to a difference in personality between the observations of Bordeaux and of Annapolis signals at the Cape. On comparing the daily differences, Greenwich–Washington, of the times of reception of the Bordeaux 20^h rhythmic signals with the corresponding differences for the Annapolis 20^h rhythmic signals, the value derived for τ was found to be $+ 0^s{\cdot}052 \pm 0^s{\cdot}002$. Concordant values were derived from (P − W), (A 20 − B 20) and from (P − W),

* Littell and Hill, *Publ. U.S. Naval Obs.*, 2nd series, vol. ix, Appendix. Difference of Longitude, Washington–Paris (1913·14); Littell and Hammond, *Ast. Journ.*, **908** (August 1928). World Longitude Operation, San Diego and Washington.

† Christie, *M.N.*, **69**. Longitude of Cape $1^h\ 13^m\ 54^s{\cdot}62 \pm 0{\cdot}022$.

‡ In this table the Paris observations are referred to the meridian of the great transit circle. Cape observations are corrected to the meridian of the old transit circle.

§ τ = time for transmission per 100° of an arc of a great circle on earth's surface.

(A 10 – B 8), viz., $0^s{\cdot}056 \pm 0^s{\cdot}002$ and $0^s{\cdot}051 \pm 0^s{\cdot}002$. When, however, the Cape observations of Bordeaux 8^h and Annapolis 10^h signals were compared with Paris and Washington observations, discordant results were derived, viz.,

from (C – P), (B 8 – A 10)	$0{\cdot}16\tau = -{\cdot}0211 \pm {\cdot}0045$
and from (C – W), (A 10 – B 8)	$0{\cdot}87\tau = +{\cdot}0795 \pm {\cdot}0051$

If the difference is assumed to arise from a difference κ of personality between the Cape observations of the two sets of signals, and $\tau = 0^s{\cdot}054$ is adopted from the above, the derived values of κ are in good agreement, viz., $0^s{\cdot}030$ and $0^s{\cdot}033$, Annapolis being observed late. Simultaneous observations by different observers in 1928 show that differences of this order may be expected.

Attributing this wholly to the Annapolis signals, the observation of which was very difficult on account of their faintness, a fresh solution was made, from which the probable error derived for unit weight was $0^s{\cdot}020$. The resulting longitudes and value of τ are

	h	m	s		s
Cape (old transit-circle)	1	13	54·582	E	± 0·0042
Paris (Cassini's meridian)	0	9	20·909	E	± 0·0016
Washington	5	8	15·753	W	± 0·0017
τ			0·054		± 0·0030

The uncertainty as to the actual distribution of personality leaves this solution no more trustworthy than the previous one.

Personal Equation in the Observation of Wireless Signals.—Since 1928 February, when the new wireless receiver was brought into use (Marconi, Type R.g. 12), with which the Rugby signal, heard only faintly at the Cape, could be observed, observations by coincidences as well as by the tape chronograph have been taken. Coincidence observations were taken from February to April by H.M. Astronomer, and from May onwards by the writer. Tape observations have been taken by Messrs. Mullis and Tibbits as well as by the writer. Comparison of the observations in monthly groups affords a determination of the difference of personality of different observers both using the tape method, as well as for two observers using different methods. In both cases the differences show a gradual change from month to month, and attain as much as $0^s{\cdot}08$, indicating a secular change in personality which reaches at least $0^s{\cdot}04$ for some observers. The comparison of the tape observations with the coincidence observations reveals a systematic difference between Rugby and Bordeaux of about $0^s{\cdot}02$, but for two observers using the tape chronograph this difference does not appear to be systematic, *i.e.* the difference (Bordeaux–Rugby) is on the whole the same for each. The order of magnitude of the probable error of a difference (tape-coincidence) for simultaneous observations is $0^s{\cdot}03$, as derived from monthly groups. This sets an upper limit to the P.E. of an observation by either method, but if it is assumed to be the same for both, the order of magnitude in each case is $0^s{\cdot}02$. The P.E. of a single night's determination of clock error is of the order $0^s{\cdot}010$,

this being the actual P.E. for a determination from observations of ten clock stars.

The value of the P.E. of a single determination of the time of reception of wireless signals at the Cape, as derived from the deviations from the means during the Longitude Programme (Table I.), is $0^s.033$. This, as may be expected from the systematic variation already noted in these two months' observations, slightly exceeds the value which would be derived from the combined errors of observation of wireless signals and clock error.

A New Spark Micro-Chronograph. By F. Hope-Jones.

Just as improved shell-resisting armour produces a bigger gun and the bigger shell is met by thicker plate, so the improvement in any one of the processes in time measurement demands a corresponding improvement in the others, and re-acts upon itself.

Until recently, unaided transit observations were good enough to check our clocks; then wireless time-signals arrived, and, by revealing discrepancies in the time determinations of distant observatories, made both look rather foolish.

The means of comparison surpassed the instrumental accuracy of clocks and telescopes. That horse shot ahead in the race and seems destined to remain in front, since wireless rhythmic signals are equal to any possible requirements. In the meantime, the introduction of the impersonal micrometer greatly improved transit observations and left the clocks a bad third.

In the last two or three years the clock has advanced and taken a position well up towards the front.

This race for accuracy demands improvement in another instrument, a more humble one, which we are apt to forget.

I refer to the chronograph on which we record the clocks against the transit observations and against one another. The method of taking time from the stars being coarse in detail though accurate in bulk, it follows that for the best results in time determination it is of great advantage to have two or more clocks compared frequently or continuously, and this comparison must be carried out with a degree of accuracy distinctly greater than that of the error it is to reveal.

Now, the maximum change of rate observed in a Shortt free pendulum clock in any one day is of the order of ·003 sec.

No ordinary chronograph is competent to handle such a small space of time. Relays and pen magnets require great care to achieve a constancy in lag of the order of ·01 sec.

Hence, when Professor Sampson began his investigation of clock performances, he armed himself with a most useful tool in the form of a *micro-chronograph*, consisting of a cinematograph film receiving oscillograph records from the clock against a tuning fork occulting a slit. This records time to within ·001 sec., but is only suitable for occasional

use. He has expressed the opinion that every observatory which seriously undertakes time determination requires such an instrument.

Hitherto chronographs have always consisted of a piece of paper or photographic film passing under a pen or a ray of light. The pen is deflected by an electro-magnet, or the ray of light by an oscillograph. The speed of the paper being known, the spaces between the marks are the measure of the time elapsing between the events. The limitations of this method are obvious ; small intervals of time can only be measured by reeling out the paper at a high speed, and that implies an enormous waste of stationery. The only continuously moving element is the paper or film.

It is the purpose of this note to describe a chronograph of an entirely different type, in which another continuously moving element is intro-

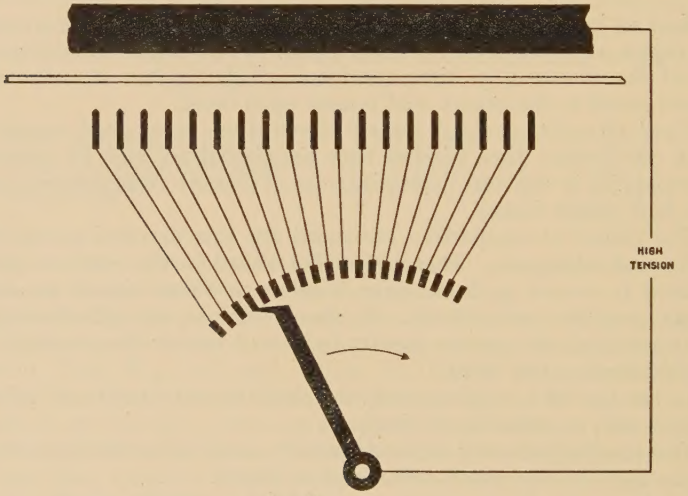

FIG. 1.

duced, in addition to the paper. This chronograph was devised and made by Mr. A. L. Loomis, in his private laboratory at Tuxedo Park, New York, where it has been running continuously day and night since the 11th of January, recording side by side on the same chart the performances of the three Shortt free pendulum clocks which he recently installed.

The chronograph propels a paper chart in the ordinary way, and the chart is unusually wide, about 10 in. It passes over a metal comb with 100 upstanding insulated teeth, each one of which is wired to a segment of a commutator (fig. 1). A rotary contact arm revolves in this commutator at a speed of 10 revolutions per second, with the result that every tooth in turn along the length of the comb provides a path for an electric current. When the Synchronome remontoire of each free pendulum operates, it makes use of whichever of these paths is open at the moment, and discharges a spark through that tooth of the comb which represents that particular one-thousandth part of

the second at which the clock has arrived. The spark pierces the paper on its way to a horizontal bus-bar immediately above it.

Thus the time-spacing is accomplished transversely across a band of paper 10 in. wide, of which every one-tenth of an inch represents the one-thousandth part of a second. The forward progression of the paper may also be considered as time-spacing, but is quite slow, about $\frac{1}{4}$ inch per half-minute, which is sufficient, since the records of each free pendulum are always at 30 secs. interval.

The regularity of the speed of rotation of the spark distributor is not of great importance when the chronograph is used merely for the purpose of comparing two or more clocks, but it would be quite another matter if it set its own dead reckoning against one clock only.

Mr. Loomis has been at great pains to secure constancy of its rotation, which is accomplished by what he calls a "super precision tuning fork." He says: "You would hardly recognise the device as an ordinary fork as it, together with its supports, weighs over 80 lb. I have checked the records of other precision tuning forks, and I think it is quite clear that the performance of this device is way beyond any ordinary fork that has ever been built. It is mounted in the constant temperature room and under a large bell jar at constant pressure. It continues day in and day out with variations of less than one part in a million."

Whilst the extraordinary accuracy of the Shortt free pendulum clock has now been established beyond any peradventure, I have never been satisfied, until now, that its half-minutes were accurately divided. Some doubts have arisen as to whether the mechanical imperfections of the Synchronome switch have not put a few extra thousandths into one second and a few less in the next, and Professor Sampson's oscillographic micro-chronograph has not resolved these doubts. Mr. Loomis's spark chronograph has apparently decided the question in favour of the clock. He writes: "The variations between successive 30-seconds readings never amount to more than ·001 of a second."

Some who have wished to use the clocks to determine radio frequencies with quartz oscillators expected to have to mount a mirror on the pendulum, reflecting a beam upon a selenium cell in order to be sure of taking a time-signal from the free pendulum with precision. It is now clear that the clock's own natural remontoire action is sufficiently precise, and it has all the merits of a substantial switch.

Proof of this has been received from an unexpected source. It will be remembered that the free pendulum has its escapement function performed for it by a slave. This slave clock is rated *slow*, and the Synchronome remontoire of the free pendulum accelerates it when necessary by the hit-and-miss synchronising spring. This spring is set to over-correct by about 100 per cent., a losing rate of about 6 seconds a day.

Mr. Loomis put the slave remontoire on to his chronograph, and got an error of ·003 sec. every other half-minute, which is exactly what would be expected. It revealed every time whether the synchroniser had hit or had missed.